LECTURE 7

Instructor's Notes

Delta and Dynamic Hedging and using Black Sholes to value companies

Review – Option Valuations - Summary

- Option values may be viewed as the sum of intrinsic value plus time or "volatility" value. The volatility value is the right to choose not to exercise if the stock price moves against the holder. Thus, option holders cannot lose more the cost of the option regardless of stock price performance.
- Call options are more valuable when the exercise price is lower, when the stock price is higher, when the interest rate is higher, when the interest rate is higher, when the time to expiration is greater, when the stock's volatility is greater and when dividends are lower.
- Options may be priced relative to the underlying stock price using a simple two-period, two-stage pricing model (Binomial Model). As the number of periods increases, the model can approximate more realistic stock price distributions.
- The Black-Scholes Model Pricing -

$$C = S\,e^{-\delta T}\,N(d1) - X\,e^{-iT}\,N(d2)$$

$$P = X\,e^{-iT}\,(1 - N(d2)) - S\,e^{-\delta T}\,(1 - N(d1))$$

Volatility is the question on the B/S –which assumes constant SD throughout the exercise period - The time series of implied volatility

THE PUT – CALL PARITY RELATIONSHIP

- Put prices can be derived simply from the prices of call
- European Put or Call options are linked together in an equation known as the Put-Call parity relationship

	St <= X	**St > X**
Payoff of Call Held	0	St – X

Payoff of Put Written	-(X – St)	0
Total	St – X	St – X

PV (x) = X e^{-rt}

The option has a payoff identical to that of the leveraged equity position; the costs of establishing them must be equal

- C – P Cost of Call purchased = Premium received from Put written
- The leverage Equity position requires a net cash outlay of S – X e^{-rt} the Cost of the stock less the process from borrowing
- C – P = S – X e^{-rt} PUT-CALL Parity Relationship - proper relationship between Call and Put

EXAMPLE:
S = $110
C = $14 for 6 months with X = $105
P = $5 for 6 months with X=$105
rf = 5.0% (continuously compounding at e)

Assumptions:
C – P = S – X e^{-rT} ?????
14 – 5 = 110 – 105 .$e^{-0.5 \times 0.5}$
9 = 7.59

This a violation of parity…. Indicates mispricing and leads to Arbitrage Opportunity

You can buy relatively cheap portfolio (buy the stock plus borrowing position represented on the right side of the equation and .sell the expensive portfolio

ARBITRAGE STRATEGY EXAMPLE:

Given the mispricing– In six months the stock will be worth Sr, so you borrow PV of X ($105) and pay back the loan with interest resulting in cash outflow of $105

Sr – 105 writing the call if Sr exceeds 105
Purchase Puts will pay 105 – Sr if the stock is below the $105

	Strategy	Immediate CF	CF if Sr < 105	CF if Sr > 105
1	Buy Stock	-110.00	Sr	Sr
2	Borrow Xe^{-iT} = $102.41	+102.41	-105	-105
3	Sell Call	14.00	0	-(Sr – 105)
4	Buy Call	-5.00	105 – Sr	0
		1,41	0	0

Whish is the difference of between 9.00 and 7.59 – riskless return

This applies if No dividends and under the European option

If Dividend then

P = C – S + PV (X) + PV (Dividend) …. Representing that the Dividend (δ) is paid during the life of the option.

<u>Example</u>

Using the IBM example – today is February 6

X = $100 (March calls)
T = 42 days
C = $2.80
P = $6.47
S = 96.14
I = 2.0%
δ = 0

P = C – S + PV (X) + PV (Dividend)   or P = C + PV (X) – S + PV (δ)

???? $6.47 = 2.80 + 100 / (1+0.02)^{42/365} - 96.14 + 0$

6.47 = 6.63 is not that valuable to go after the repricing arbitrage

PUT OPTION VALUATION

$P = X e^{-iT} (1- N (d2)) - S e^{-\delta T} (1 - N (d1))$

Using the data from previous example

$P = 95. e^{-10x.25} (1 – 0.05714) – 100 (1 – 0.6664)$

P = 6.35

PUT-CALL Parity

P = C + PV (X) – So + PV (Div)

$\backslash P = 13.70 + 95 . e^{-10 X 0.25} – 100 + 0$

<u>Hedge Ratios & the B/S format</u>

The Hedge ratio is commonly called the **<u>Option Delta</u>**. Is the change in the price of call option for $1 increased in the stock price

This is the slope of value function evaluated at the current stock price

For Example

Slope of the curve at S = $120 equals .60. As the stock increases by $1, the option increase on 0.60

For every Call Option Written, .60 shares of stock would be needed to hedge the Investment portfolio.

For example, if one writes 10 options and holds 6 shares of stock,

H = .60 ……… a $1 increase in stock will result $6 gain ($1x 6 shares) and with the loss of $6 on 10 options written (10 x $0.60)

- The Hedge Ratio for a Call is N (d1),

- with the hedge ratio for a Put [N (d1) – 1]
- N (d) is the area under standard deviation (normal)
- Therefore, the Call option Hedge Ratio must be positive and less than 1.0
- And the Put option Hedge Ratio is negative and less than 1.0

Example:

2 Portfolios

Portfolio	A	B
BUY	750 IBM Calls 200 Shares of IBM	800 shares of IBM

Which portfolio has a greater dollar exposure to IBM price movement?

Using the Hedge ratio you could answer that question:

Each Option change in value by H dollars for each $1 change in stock price

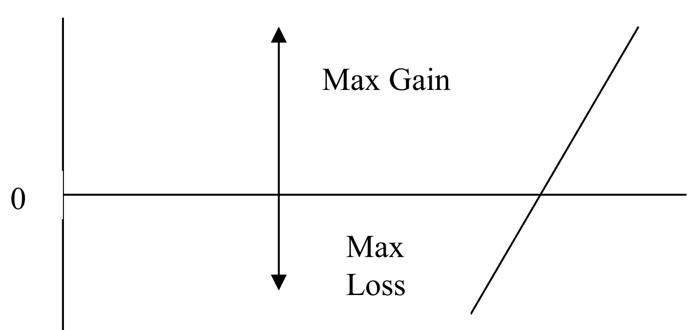
If H = 0.6, then 750 options = equivalent 450 shares (0.6 x 750)

Portfolio A = 450 equivalent + 200 shares which is less than Portfolio B with 800 shares

PORTFOLIO INSURANCE (PROTECTIVE PUT STRATEGY)

MAX LOSS:
At the money (X = S) the maximum loss than can be realized is the cost of the Put

MAX GAIN
Unlimited (sale of the stock)



- p ________________________

Desired horizon of the Insurance Program must match the maturity of a traded option in order to establish the appropriate put positions

Most options don't go over 1 year – There is an Index options LEAPS (Long Term Equity AnticiPation Securities)

Synthetic Protective Puts – gives you a hedge mechanism without buying an option.

<u>Example– Chasing the Deltas – Synthetic Protective Puts</u>

Portfolio of $100 million
At the money Put Option on the portfolio with a Hedge ratio or Delta = 0.06 which means the option value swings $0.60 for every $1 change in the opposite direction

Portfolio goes DOWN by 2%

The profit of the hypothetical protective Put position (if the put existed) will be as follows:

Loss on stock	2.% x 100 =	$2.0 million
Gain on Put	.6 x 2.00 =	$1.2 million
Gain (loss)		$0.8 loss

We created the synthetic option position by selling a proportion of shares equal to the put option or Delta… ie. Sells 60% of the shares and the proceeds are placed in rf (risk free rate) T-Bills –

DYNAMIC HEDGING – Constant updating of **<u>hedge positions as market conditions change</u>**

A SIMPLE EXAMPLE OF DYNAMIC HEDGING

To start off, consider the following example, which we have adapted from Hull (1997): A financial institution has sold a European call option for $300,000. The call is written on 100,000 shares of a non-dividend paying stock with the following parameters:

Current stock price = $49
Strike price = X = $50
Stock volatility = 20%
Risk-free interest rate r = 5%.
Option time to maturity T = 20 weeks
Stock expected return = 13%

The Black-Scholes price of this option is slightly over $240,000.

It follows that the financial institution has earned approximately $60,000 from writing the call. However, unless the financial institution *hedges* its obligation in order to offset the effects of price changes in the future, it could stand to lose much money at the call's expiration. For example, if the price of the stock at the option's expiration date is $60, and if the call is not hedged, then the financial institution will lose $1,000,000 (=100,000*(60-50)) at the option's expiration.

<u>Strategies include:</u>

1. Stop-Los Strategy
2. Delta Hedging

APPLICATIONS OF OPTION PRICING THEORY TO EQUITY VALUATION

- Most equity analysts do not associate option pricing theory with equity or asset valuation.
- This Lecture will aim to show that option pricing theory has an important role o play in valuation and that it provides a very different perspective that can be useful in understanding and analyzing troubled firms, natural resource firms, and high-technology firms.

<u>Valuing Equity as an option</u>

- Traditional methods for valuing equity include DCF, Trading and Acquisition Multiples,

- The equity in a firm is a residual claim – that is, euity holders lay claim to all cash flows over after financial claimholders (debt, preferred stock) have been satisfied.
- The payoff to equity investors, on a liquidation basis (bottom of the waterfall) can therefore be written as:

Payoff to equity on liquidation = EV – D if EV > D
and = 0 if EV ≤ D

where EV = Enterprise Value and D = Debt

- A Call option with strike price (X) on an asset with a current value of stock (S) has the following payoffs:

Payoff on Exercise = S – X if S > X
And = 0 if S ≤ X

Equity can thus be viewed as a call option on the firm, where exercising the option requires that the firm be liquidated and the face values of the debt (which corresponds to the exercise price) be paid off.

Example

- Asset Value / Enterprise Value of the firm = $100 million
- Standard Deviation in the asset value = 40%
- Face Value of the Debt = $80 million (10 year – zero coupon debt at 10%) – (estimated PV of $80 million at 10% for 10 years = $30.84 not using e compounding)

How much is the equity worth today?
What should the interest rate on debt be?

The use of option pricing approach provides a solution:

- Value of underline asset = S = Value of the firm = $100 million
- Exercise Price = X = Face Value of outstanding Debt = $80 million
- Life of the option = t = Life of zero-coupon debt = 10 years

- Variance in the value of the underlying asset = σ^2 = Variance in EV = $.4^2$ =.16
- Riskless Rate = i = 10 year T-Bonds rate to option life = 10%

d1 = 1.5994 and d2 = .3345
N(d1) = .9451 and N(d2) = .6310

Value of the Call using Black-Scholes:

(100 x .9451) – ($80 $e^{-.10x.10}$ x .6310) = $75.94 million

Value of outstanding debt = $100 - $75.94 = $24.06 million

Increase rate on debt = ($80 / $24.06) $^{1/10}$ -1 = **12.77%**

Valuing Equity in a Troubled Firm using Option Pricing

- The first implication is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt
- Such a firm will be viewed as troubled by investors, accountants and analysts, but that does not mean that it's equity is worthless.
- There is a possibility that the underlying value might increase by expiration day.

Example – (using the example above)

- Asset Value / Enterprise Value of the firm = $50 million
- Standard Deviation in the asset value = 40%
- Face Value of the Debt = $80 million (10 year – zero coupon debt at 10%) – (estimated PV of $80 million at 10% for 10 years = $30.84 not using the e compounding)

The $50mm is well below the $80 million outstanding debt (due 10 years from now)

The use of option pricing approach provides a solution:

- Value of underline asset = S = Value of the firm = $50 million

- Exercise Price = X = Face Value of outstanding Debt = $80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in EV = $.4^2$ =.16
- Riskless Rate = i = 10 year T-Bonds rate to option life = 10%

d1 = 1.0515 and d2 = - .2135
N(d1) = .8534 and N(d2) = .4155

Value of the Call using Black-Scholes:

(50 x .8534) – ($80 $e^{-.10x.10}$ x .4155 = $30.44 million

Value of outstanding debt = $50 - $30.44 = **$19.56 million**

The Equity in this firm has substantial value, because of the option characteristics of equity. This might explain why stock in firms that are in "Chapter 11" and essentially bankrupt still has value.

The Conflict between Bondholders and Stockholders

- Different objectives between Stockholders and Bondholders (Risk)
- This conflict can be illustrated dramatically using the option pricing model.

Example – (using the example above)

- Asset Value / Enterprise Value of the firm = $100 million
- Standard Deviation in the asset value = 40%
- Face Value of the Debt = $80 million (10 year – zero coupon debt at 10%) – (estimated PV of $80 million at 10% for 10 years = $30.84)

- Value of the Equity = E = \$74.94 million
- Value of the Debt = D = \$24.06 million
- Value of the Firm = EV = \$100 million

Now assume that the stockholders have the opportunity to take a project with negative net present value of - \$2 million, but assume that this project is very risky project that will push up the standard deviation in firm value to 50%. The equity as a call option can then be valued using he following inputs:

- Value of underline asset = S = Value of the firm = \$98 million (100-2)
- Exercise Price = X = Face Value of outstanding Debt = \$80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in EV = $.5^2 = .25$
- Riskless Rate = i = 10 year T-Bonds rate to option life = 10%

Based on Black-Scholes:

- Value of the Equity = E = \$77.71million
- Value of the Debt = D = \$20.29 million
- Value of the Firm = EV = \$98 million

The value of equity rises from \$75.94 million to \$77.71 million, even though the firm value declines by \$2 million. The increase in equity value comes at the expense of bondholders, who see their wealth decline from \$24.06 million to \$20.19 million.

Valuing Equity as an Option: Wang Labs

File for Bankruptcy 1993

DEBT ASSUMPTIONS			**VALUE ASSUMPTIONS (Pre-bankrupcy)**	
Debt Outstanding =	529.4		Stock Montly Var. (1987 - 1991) =	0.0262
Weighted Average Duration=	5.1	years	Bonds Monthly Var. (1987 - 1991) =	0.0126
Weighted Average maturity=	8.7	years	Correlation between Stock/Bond	0.27
WACC=	10%		Debt proportion (1987 - 1991) =	86.10%
Tax Rate =	36.0%			

Discount Cash Flow Analysis		**1993**	**1994**	**1995**	**1996**	**1997**
Revenue		1,300.0	1,010.0	1,067.0	1,121.0	1,177.0
CoGS		(1,000.0)	(658.0)	(705.0)	(741.0)	(778.0)
Oper. Exp.		(750.0)	(279.0)	(267.0)	(269.0)	(282.0)
EBIT		**(450.0)**	**73.0**	**95.0**	**111.0**	**117.0**
EBIT (t)		**(162.0)**	**26.3**	**34.2**	**40.0**	**42.1**
EBIT (i-t)		**(288.0)**	**46.7**	**60.8**	**71.0**	**74.9**
Less Capex (offset by Depreciation		-	-	-	-	-
Less W/C		-	-	-	-	-
Cash Flow		(288.0)	46.7	60.8	71.0	74.9
Terminal Value assumption	6.8x					794.0
EV (PV) of the firm	**$410.5**	**(288.0)**	**46.7**	**60.8**	**71.0**	**868.9**

Step 1 - Find the annualized in stock and bond prices:

Annualized Variance in Stock Price σ^2 =	0.3144	(annual)	St. Dev.=	0.560714
Annualized Variance in Bond Price σ^2 =	0.1512	(annual)	St. Dev.=	0.388844

Step 2 - Find the annualized variance in firm value

(we^2 x σe^2) + (wb^2 x σb^2) + (we x wd x ped x σe x σd)

We= 13.90%
Wd= 86.10%

Annualized Varaince in firm value 0.132253

The five-year bond rate (corresponding to the weighted average duration of 5.1 years) is 6.0%

Stet 3 - Find the value of call based upon the following parameters of equity as a call option

Value of the underlying asset = S = Value of the firm =	$410.5
Exercise Price = X = Face Value of outstanding debt =	$529.4
Life of the option = t = Weighted average duration of debt=	5.1 years
Variance in the value of the underlying asset = σ^2 =	0.132253
Riskless Rate = I = T-Bond for option life =	6.00%

d1=	0.473506	N (d1) =	0.682074
d2=	-0.347767	N (d2) =	0.364008

Value of the call = 137.9534

Wang's equity was trading at $85 milion in March 1993

CASE STUDY: Airline Company

NYSE Stock Standard Deviation=	25%	
Debt average % o EV=	90%	
Bond Rating =	B	bonds are not traded
Other B ratednames Stand Dev=	10%	
Correlation Stock/Bond market	0.3	
Dividends =	0	
T-Bond	8.00%	

Enterprise Value	
North America	400
Europe	500
South America	100
	1000

Debt	**Amount**	**% Total**	**Coupon**	**Duration**	**WA Dur**
20-yr debt	100	8.33%	11.0%	14.1	1.1750
15-yr debt	100	8.33%	12.0%	10.2	0.8500
10 yr debt	200	16.67%	12.0%	7.5	1.2500
1-yr debt	800	66.67%	12.5%	1.0	0.6667
Total Debt	1200	100.00%			3.9417

Variance of the Firm	% W	σ	W^2	σ^2	W^2xSD^2	We.Wd.σe.σd
Equity % of Cap	10%	25%	0.01	0.0625	0.00063	
Debt % of Cap	90%	10%	0.81	0.01	0.00810	
					0.00873	0.00225
					Variance =	0.01098
					Stand Dev=	0.104761634

Value of Equity using Option Priocing (Black-Scholes)

		Ln (EV / Debt)	(i + σ^2/2)*t	σ?t	d1 and d2
d1=	0.7435	-0.182321557	0.34	0.21	0.7435
d2=	0.5355				0.5355
N(d1)=	0.7714				
N(d2)=	0.7039				
Value of the Call=	**154.63**				