Return Basics – Time Value of Money

Risk & Return Analysis

- Before investing, the investor needs to consider the following four factors:
  1. Measurement of the expected return (Er)
  2. Quantification of the risk (σ)
  3. How to allocate the investments to achieve efficiency and optimization (A for Allocation)
  4. Time to determine the exit strategy or realization of the investment (t)

Time Value of Money Concepts

- One-Time Investment
- Annuities or Even Annual Cash flows
- Uneven Annual Cash Flows

One-Time Investment

FUTURE VALUE

$$FV = PV\,(1 + i)^t$$

where FV is the future value of the investment, PV is the present value of the investment or the initial investment, i is the expected interest rate or rate of return of the investment, and t is time to realize such investment.

Future Value – Example 1

Suppose a relative of yours deposited $10 at 5% interest into a savings account 200 years ago. How much would the investment be worth today with simple interest? With compound interest? What is the difference?

Simple 10 + 10 (0.5) (200) = 110

Compound FV = 10 (1+1.05) ^ 200 = 172,925.81

Future Values – Example 2

- Suppose you invest the $1,000 at an interest rate of 5% for 10 years. How much would you have assuming compound interest?
  - 1,000 * (1.05) ^10 = $1,628.89
  - What is the future value assuming simple interest?
  - 1000 * (.05)(10) = 1,500 _

What is the difference between earning compound and simple interest?

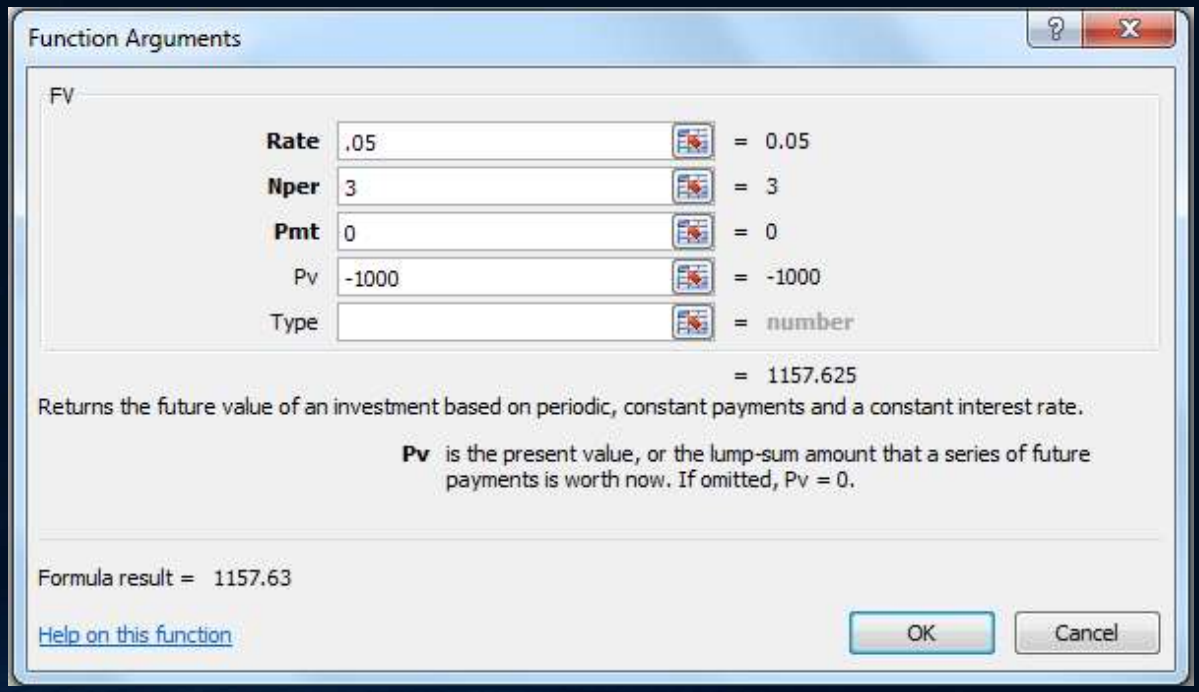
- 128.89

FV: Using Excel's Financial Technology

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > FV.
  - Fill in the Rate, number of periods (Nper), periodic payments (Pmt), and present value (Pv). Click OK.
- Type the function
  - =FV(rate, nper, pmt, pv, type)

FV: Excel Dialogue Box Technology

- If you invest $1000 for 5% per year how much money will you have in 3 years?



FV: Excel Formula

Investment	= CF_0 =	-$1,000.00				
Interest rate	= I =	5.00%				
No. of periods	= N	3				
		Periods:	0	1	2	3
		Cash Flow Time Line:	-$1,000			FV = ?
Step-by-Step Approach:			$1,000	$1,050.00	$1,102.50	$1,157.63
Formula Approach: $FV_N = PV(1 + I)^N$			FV_N =	$1000(1.05)^3$	=	$1,157.63
Excel Approach:	FV function:	FV_N =	=FV(rate, nper, pmt, pv, type)			
	Fixed inputs:	FV_N =	=FV(0.05, 3, 0, -1000)	=	$1,157.63	
	Cell references:	FV_N =	=FV(C3, C4, 0, C2)	=	$1,157.63	

Future Value – one more example

A company currently has profits of $3 million dollars. If profits increase 4% each year, how much profit will the company earn in 10 years?

3mm , i=0.04 t = 10

FV = 3 (1.04) ^ 10

Future Value – another example

You borrow $10,000 in student loans your freshman year at an annual interest rate of 6.2% per year. If you have a grace year and don't have to start paying back the loan until after you graduate, how much money do you owe 5 years later?

PV – 10,000, i=0.62, t= 5

10,000 * (1.062) ^5 = 13,508.98

Future Value – One more example

If a stock which is currently worth $32 is expected to have return 8% annually, how much will it be worth in 5 years?

Pv = 32

I = 8.0%

47.02

One-Time Investment

PRESENT VALUE

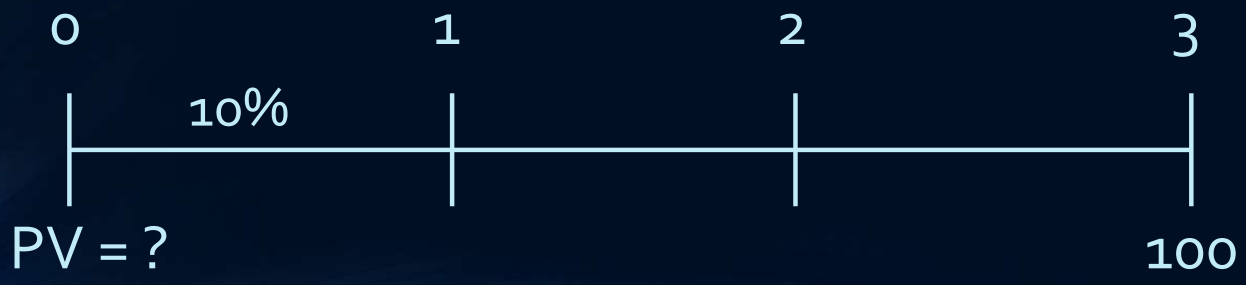
$$FV = PV\,(1 + i)^t, \textit{ then } PV = \frac{FV}{(1+i)^t}$$

As an example, assuming the investor targets an investment that is expected to receive $133.10 in 3 years, representing a 10% interest or expected return (sometimes referred to as the discount rate), the investment required today will be calculated as follows:

- $PV = \frac{FV}{(1+i)^t} = \frac{133.10}{(1+.10)^3} = \frac{133.10}{1.331} = 100$

Present Value

- The amount an investment is worth today (now, at time 0).
- Finding the PV of a cash flow or series of cash flows is called discounting.
- The value of cash flows in today's purchasing power.



Present Values

- We can use the future value formula to find the present value.
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- To find present values

PV: The Formula Method

How much money must you invest today if you can earn 7% and you would like to have $5,000 in 10 years.

75.13

Present Values – Example 2

Suppose when you were born, your parents wanted to begin saving for your college education and they estimated that after 17 years you would need $150,000. If they felt confident that they could earn 8% per year, how much did they need to invest when you were born?

150,000 , 17 years 8.0% = 40,540.34

Present Values – Example 3

You win a raffle and have the option of two prizes. You can either receive $1000 now or $1075 at the end of the year. If the discount rate is 8%, which prize do you prefer?

PV = 1075 / 1/08 < 1000

FV = 1000 (1.08) > 1075

1,075 / (1+ .08)

PV: Using Excel's Financial Technology

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > PV.
  - Fill in the Rate, number of periods (Nper), periodic payments (Pmt), and future value (Fv). Click OK.
- Type the function
  - =PV(rate, nper, pmt, fv, type)

Effects of Compound Interest

- For a given interest rate – the longer the time period, the lower the present value.
- For a given time period – the higher the interest rate, the smaller the present value

Present Value

How much money must you invest in an account that earns 8% today if you want to have $1000 in 5 years?

PV = FV / (1+i) ^ t

PV = 1000 / (1.08) ^ 5 = 680.53

Present Value

How much does a car cost in today's dollars if the dealer offers you a no money down 12% loan which requires full payment of $42,296.20 in 5 years?

PV = 42,296.20 / (1.12)^5 = 24,000

One-Time Investment

INTEREST RATES

If the investor knows the amount they are planning to invest today, and targets a specific investment payoff at a set time in the future, then the investor can rearrange the formula to calculate the interest (i) or discount rate that he or she will earn, as follows:

Starting at $FV = PV\ (1 + i)^t$, *then* $(1 + i)^t = \frac{FV}{PV}$, and

$$\mathbf{i = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1}$$

As an example, let's assume the investor invests $100 today and targets an investment that expects to receive $133.10 in 3 years. What will the annual rate of return be on such an investment?

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1 = \left(\frac{133.10}{100}\right)^{\frac{1}{3}} - 1 = (1.331)^{\frac{1}{3}} - 1 = 1.10 - 1 = 0.10 = 10\%$$

Simple Interest – Example 1

- FV with simple interest

  The $1,000 investment accrues $50 each year for each of two years, So in 2 years you have:
  - FV = 1000 + 50 = 1050
  - 1050 + 50 = 100

- What is the future value of $1,000 after 2 years if the interest rate is 5% compounded annually?

  1,000 * (1.05)*(1.05) = 1000 * (1.05)^2 = 1102.50

Interest Rate

- Suppose you buy a stock for $30 and sell it for $33 at the end of the year. How much did you earn? Amount? Rate?

- 33 – 30 = 3
- 3/30 = 10%

Discount Rate – Example 1

You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. How much interest does this investment earn?

FV = PV (1+i)^t

33 = 30 (1+i)^1

33/30 = 30 (1+1+i) / 30

33/30 – 1= 10.0%

Discount Rate Formula

- Start with basic equation and solve for r
  - $FV = PV(1 + r)^t$

Discount Rate

$$r = \left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{\frac{1}{t}} - 1$$

Discount Rate – Example 2

Suppose you have a 1-year old son and you want to provide $75,000 in 17 years toward his college education. You currently have $5,000 to invest. What interest rate must you earn to have the $75,000 when you need it?

R = (fv / pv) ^ 1/t – 1 = 17.27%

Discount Rate: Using Excel

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > RATE.
  - Fill in the, number of periods (Nper), periodic payments (Pmt), present value (pv), and future value (Fv). Click OK.
- Type the function
  - = RATE(nper, pmt, pv, fv, type, guess)

Interest Rate

Suppose you buy a home for $300,000 and sell it 5 years later for $400,000. What yearly interest rate did you earn?

PV = 300k, FV = 400k t = 5 I = (400/300) ^ 1/5 – 1 = 5.92%

Interest Rate

If you buy a stock for $32 and sell it for $33.60 one year later, what yearly interest rate did you earn?

I = 33.60 /32) ^1/1 – 1 = 5.0%

One-Time Investment

TIME

If the investor knows the amount that they are planning to invest today, then sets a target payoff amount in the future and assumes a given rate of return, then he or she can calculate how long it will take to achieve the target. The time (t) to realize the targeted return is calculated by rearranging the formula as follows:

Starting at $FV = PV\ (1 + i)^t$, *then* $(1 + i)^t = \frac{FV}{PV}$, then adding ln on both sides, you get $\ln\ (1 + i)^t = \ln(\frac{FV}{PV})$, *t* $[\ln(1 + i)] = \ln(\frac{FV}{PV})$, *and*

$$t = \frac{ln\ (\frac{FV}{PV})}{ln(1+i)}$$

- As an example, assume the investor invests $100 today and wants to find out how long it will take for the investment to reach $133.10 if invested at an annual rate of return of 10%. The time to reach the targeted future value of such investment is calculated as follows:
- $t = \frac{ln\ (\frac{FV}{PV})}{ln(1+i)} = \frac{ln\ (\frac{133.10}{100})}{ln(1+0.10)} = \frac{ln\ (\frac{133.10}{100})}{ln(1+0.10)} = \frac{ln\ (1.331)}{ln(1.100)} = \frac{0.2859}{0.0953} = 3$ years

Number of Periods – Example 1

You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?

FV = $20,000, r = 10%, PV=15,000 t=?

FV = PV (1+i) ^t

20/15 = 1.1^t

Ln(20/15) = ln (1.i)^t) ln (20/15)/ ln (1.1) = ln 1.1 / ln (1.1) = 3.02 years t =

Number of Time Periods

- Start with basic equation and solve for t (remember your logs)
  - $FV = PV(1 + r)^t$

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + r)}$$

Number of periods

You buy a stock for $25 which you expect to grow at a rate of 7% each year. How long will it be before the stock is worth $50?

PV = 25

FV = 50

I = 7.0%

T = ?

T = Ln (FV/PV) / ln (1+i) = ln (50/25) / ln (1.07) = 10.24 years

Number of Periods – Example 2

If you can earn 10% annual interest, how long does it take for your investment to double?

Pv = 1

FV = 2

7.27 years rule of 72

Number of Periods: Using Excel

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > NPER.
  - Fill in the Rate, periodic payments (Pmt), present value (pv), and future value (Fv). Click OK.
- Type the function
  - =NPER(rate, pmt, pv, fv, type)

Annuities or Even Annual Cash flows

FUTURE VALUE

$$FVA = CF + CF(1 + i) + CF(1 + i)(1 + i), \text{ or } FVA = CF\left(\frac{(1+i)^t - 1}{i}\right)$$

For example, if an investor invests $100 per year for 3 years and expects a 10% rate of return, then the value of such investment when it is cashed out in 3 years will be calculated as follows:

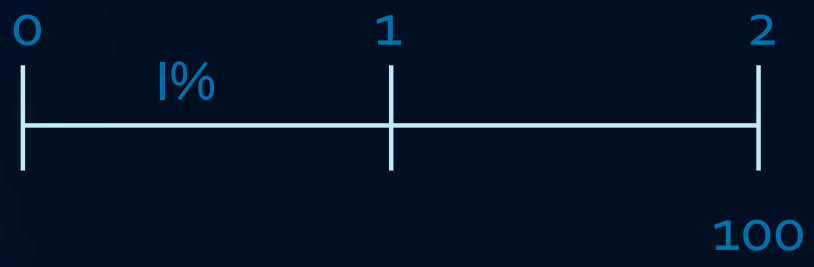
$$FVA = CF\left(\frac{(1+i)^t - 1}{i}\right) = 100\left(\frac{(1+0.10)^3 - 1}{.10}\right) = 100\left(\frac{(1.10)^3 - 1}{.10}\right) = 100\left(\frac{1.331 - 1}{.10}\right) = 331.00$$

Annuities

- Annuity – a finite series of equal payments that occur at regular intervals
- Examples

Drawing Timelines

$100 lump sum due in 2 years



3 year $100 ordinary annuity

FV of an Ordinary Annuity - Example

How much will an ordinary annuity be worth in 3 years, if it pays $100 per year and earns 10% annual interest?

100 , 100 , 100

FV = ? 100 (1.1)^2 + 100 (1.1) + 100 = 331

FV of an Ordinary Annuity: Formula

$$FVA_t = \sum_{t=1}^{N} CF \cdot (1+r)^{N-t}$$

$$FVA_t = CF\left[\frac{(1+r)^t - 1}{r}\right]$$

FV of an Ordinary Annuity - Example

How much will an ordinary annuity be worth in 3 years, if it pays $100 per year and earns 10% interest? (Use the formula)

CF = 100, t = 3 years , FV =

I = 10%, PV = 0

FV = CF ((1+i)^t) -1) / r

FV = 100 ((1+i)^2 – 1) / 0.10

FV = 331

FV of an Ordinary Annuity - Example

Payment amount	$100.00
Interest rate	10.00%
Number of periods	3

Cash Flow Time Line: -$100 -$100 -$100

Step-by-Step Approach:

Multiply each payment by $(1+I)^{N-t}$ and sum these FVs to find FVA_N:

-$100.00	
-110.00	
-121.00	
-$331.00	

Excel Function Approach:

FV function:	FVA_N =	=FV(rate,nper,pmt,pv,type)	
Fixed inputs:	FVA_N =	=FV(0.1,3,-100,0) =	$331.00
Cell references:	FVA_N =	=FV(C132,C133,-C131,0) =	$331.00

Annuity (Real-world) Example

You decide that starting when you are 20 years old you will save $3 a day for retirement. Every day you put $3 in a drawer. At the end of the year, you invest the accumulated savings ($1,095) in a brokerage account with an expected annual return of 12%. If you continue the practice every year until you are 65, how much money will you have?

CF = 10,095 , FV = ? , T= 45 years, PV = 0

FV = CF ((1+r)^t – 1) / r

FV = 1095 ((1+.12)^45 – 1 / .12

$1,487,261.89

More Examples

- If you invest $100 at 5% interest how much money will you have in 8 years?
- FV = PV (1+r)^t = 100 (1.05)^8 = $147.75

- If you invest $100 per year at 5% interest, how much money will you have in 8 years?

- FV = CF [((1+i)^t) – 1]/ r = 100 ((1.05)^8 – 1) / 0.05 = $954.91

Annuity - Cash Flow

If you would like to have $120,000 in a your child's college account in in 18 years and you can earn 6.5% per year, how much must you invest each year?

FV = 120,000, t=18 years, r = 6.5%, PV = 0 , CF = Pmt ?

FV = CF ((1+i)^t) – 1) / r

120,000 = CF ((1.065)^18 – 1) / 0.065

CF = $3702.55

Who wants to be a millionaire?

How much money do you need to save each year to be a millionaire by the time you are 65 if you can earn an interest rate of 8% and you start saving when you are

45? $21,852.21
30? $5,803.26
20? $2,587.28
Born? $451.35

FV=$1 mm
PV = 0
t =65-45 = 20 years
R = 8.0%
=pmt (

Annuity - Time

You want to buy a $40,000 antique car. If you contribute $1200 per year to a savings account which earn 11% interest how long will it take before you can buy the car?

FV = $40,000 , PMT = 1,200 , r=11%, t=?

FV = 40,000 = 1,200 (1.11)^t – 1 / .11

t= ? 14.75 years

NPER = (

Annuities or Even Annual Cash flows

PRESENT VALUE

The present value of an annuity (PVA) can be calculated as follows:

$$\text{PVA} = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \ldots\ldots + \frac{CF}{(1+i)^n}, \text{ or PVA} = \mathbf{CF}\left[\frac{1-\frac{1}{(1+i)^t}}{i}\right]$$

For example, if an investor expects to receive $100 per year for 3 years, then what is the present value for an investment if the investor expects to receive a 10% annual rate of return? The calculation of the present value of such an investment is as follows:

$$\text{PVA} = \text{CF}\left[\frac{1-\frac{1}{(1+i)^t}}{i}\right] = 100\left[\frac{1-\frac{1}{(1+.10)^3}}{.10}\right] = 100\left[\frac{1-\frac{1}{1.331}}{.10}\right] = 248.69$$

Uneven Annual Cash Flows

- If the investment expected to produce uneven annual cash flows to the investor, called payments, for a set time, using the same expected rate of return, then the investment is calculated differently. The present value of such cash flows is the sum of all the future cash flows discounted back at a given expected rate of return, as follows:
- PV = $\frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \cdots \frac{CF_t}{(1+i)^t}$ $\ldots . PV = \sum \frac{CF_t}{(1+i)^t}$
- For example, if an investor expects to receive $95 the first year, $92 the second year, and $105 the third year, what is the present value for such an investment if the investor expects a 10% annual rate of return? The calculation of the present value of such investment is as follows:
- PV = $\frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} = \frac{95}{(1+0.10)^1} + \frac{92}{(1+1.10)^2} + \frac{105}{(1+1.10)^3} =$
- $86.36 + 76.03 + 78.89 = 241.28$

Annuity - Time

You want to buy a $40,000 antique car. If you contribute $1200 per year to a savings account which earn 11% interest how long will it take before you can buy the car?

t= ? 14.75 years

Annuity – Interest rate

What interest rate must you earn in order to have $500,000 in a savings account in 20 years in you contribute $2400 per year?

R = ? , PV = 0, FV = $500,000 , t=20 , CF = $2,400

FV = CF (l1+r)^t – 1) / r

USE EXCEL

20.90%

Discount Rate: Using Excel

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > RATE.
  - Fill in the, number of periods (Nper), periodic payments (Pmt), present value (pv), and future value (Fv). Click OK.
- Type the function
  - = RATE(nper, pmt, pv, fv, type, guess)

Annuity – Interest rate

HPR vs Interest Rate same as IRR

What interest rate must you earn in order to have $500,000 in a savings account in 20 years in you contribute $2400 per year?

=Rate (nper, pmt, pv, fv, type, guess)

20.90%

Present Value of an Annuity

How much is an ordinary annuity **currently** worth, if it pays $100 per year for 3 years and earns 10% interest?

$248.68 = 100 /(1.1)^1 + 100 / (1.1)^2 + 100 / (1.1)^3 =

Cf / (1+r)^1, CF /(1+r)^2

PV of an Annuity: Formula

$$PVA_t = \sum_{t=1}^{N} \frac{CF}{(1+r)^t}$$

$$PVA_t = CF\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

PV of an Ordinary Annuity - Example

How much is an ordinary annuity currently worth, if it pays $100 per year for 3 years and earns 10% interest? (Use the formula)

PV = CF (1 – ((1 / (1+r) ^ t)) / r

PV = 100 (1 – (1/(1+.1)^3) / 0.10

PV = $248.68

Present Value of an Annuity

How large a mortgage can you get if you can afford payments of $8000 per year for the next 30 years and the interest rate is 4.68%?

PMT = CF = $8,000 , t = 30, r=4.68%, PV = ?

PV = CF (1 – 1 / (1+r)^t) / r

PV = 8,000 (1- 1 /(1+0.0468)^30) / 0.0468

PV = $127,595.80

PV: Using Excel Technology

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > PV.
  - Fill in the Rate, number of periods (Nper), periodic payments (Pmt), and future value (Fv). Click OK.
- Type the function
  - =PV(rate, nper, pmt, fv, type)

Retirement

After you retire, you want to withdraw $50,000 a year from your savings account which earns 4% annual interest for 30 years. How much money do you need in your saving account when you retire?

PV = ?, CF = $50,000 , t = 30 , r =4.0%, FV = 0

PV = 50,000 (1- 1/(1+.04)^30) / 0.04

PV = $864,601.67

Annuity – Time

If you borrow $60,000 in student loans at an interest rate of 7.5% and you can afford to pay $4,800 per year how long will it take you to pay off your loans?

t = ?, PV = $60,000 , Pmt = $4,800 , r = 7.5%

60000 = 4,800 (1-(1/(1.075)^t / 0.075

T = 38.34 years

Annuity – Interest rate

At retirement you have $750,000 in your savings account. You intend to withdraw $65,000 per year. What interest rate must you earn to make your money last for the next 30 years?

PV = $750,000, PMT = $65,000, t = 30, FV = 0, r = ?

Rate = ? = 7.74%

Annuity - Recap

- An annuity is a financial security which consists of a finite series of equal payments that occur at regular intervals.
- You can find the future value of an annuity by:
  - Adding the future values of each payment.
  - Using the formula.
  - Using Excel.

Excel based formulas:

Excel formulas for calculating all five variables including the present value, future value, rate of return, time and cash flows or payments (represent set additional payments received during the investment):

= PV (rate, years, payment, future value) or *=pv(rate,nper,pmt,fv)*

=FV (rate, years, payment, -present value) or *=fv(rate,nper,pmt,pv)*

=Rate (years, payment, - present value, future value) or *=rate(nper,pmt,pv,fv)*

=Nper (rate, payment, - present value, future value) or *=nper(rate,pmt,pv,fv)*

= Pmt (rate, years, -present value, future value) or *=pmt(rate,nper,pv,fv)*

Measuring Return and Return Expectation

- Before you invest your money in any securities or any businesses, it's extremely important to consider and must measure the following four factors:
  1. **Return expectation**
  2. **Risk**
  3. **Allocation**
  4. **Time**

Objective of Return and Return Expectation

- As discussed previously, it is essential to measure the historical returns of a particular asset class and then set an expectation going forward based on various methods that we will examine in this section. The return analysis objectives are as follows:

  - **Trend Analysis**: To compare the return analysis to historical trends of the particular investment, project the trend going forward, and adjust such trends based on views for driving such investment
  - **Comparative Analysis**: To compare it to other asset classes and/or the market, and/or the risk-free rate.
  - **Expectation Analysis:** To compare it to last year's expectations and continue to test the performance versus expectation

Rates of Return: Holding Period Return (HPR)

$$\text{HPR} = \frac{\text{CF}}{\text{I}}$$

where CF is the cash flow (inflow and outflow) during the investment period and I is the initial investment.

For example, if an investor buys the stock for $100 and sells it for $120 and during the investment he or she received $2 in dividends, then the cash flow on the numerator is $120 of proceeds for selling the stock plus $2 of cash dividends received (cash inflow) minus the initial investment of $100 (cash outflow), and the net cash flow is $22 = ($120 + $2 - $100). The HPR will be then be calculated by dividing the net cash flow of $22 by the initial investment of $100 resulting in a 22% return:

$$\frac{(120 - 100 + 2)}{100} = \frac{22}{100} = 0.22 = 22\%$$

Rates of Return:
Annual Rate of Return (ROR) and Internal Rate of Return (IRR)

The more challenging calculation is if the payment is different every year, so the annual rate of return must be weighted based on size and the year paid sometime referred to as dollar weighted return. This type of rate return method is the internal rate of return. It's challenging because each year the investment would have different payoffs and sometimes negative numbers. The best approach to calculate the IRR is using spreadsheet analysis. The formula on excel is as follows:

$=IRR(CF_0, CF_1, CF_2, CF_3 \ldots CF_t)$.

Rates of Return: Annual Rate of Return (ROR) and Internal Rate of Return (IRR)

INTERNAL RATE OF RETURN (IRR)

Dollar Weighted Return (Even Annual Payments)

	0	1	2	3	4
Net CF ($)	-100	5	5	5	105

EXCEL

	A	B	C	D	E	F
10	IRR	0	1	2	3	4
11	5.00%	-100	5	5	5	105

=IRR(B11:F11)

Dollar Weighted Return (Uneven Annual Payments)

	0	1	2	3	4
Net CF ($)	-100	-9	-5	26	110

$$1 = + (-0.1 / (1+IRR) + (-0.5 / (1+ IRR)\ ^2 + (0.8 / (1+ IRR)^3) + (1.0 / (1+IRR)^4$$

EXCEL

	A	B	C	D	E	F
10	IRR	0	1	2	3	4
11	4.96%	-100	-9	-5	26	110

=IRR(B11:F11)

Figure 1.1

Measuring Return and Quantifying Risk

- **Rate of Return: Average Annual Rate of Return**

  The average annual rate of return can be calculated by looking at 5 to 10 years of historical returns and averaging them to represent the annual return. It gives the investor an indication what to expect on an average. This method is used for comparing to other investments that were calculated the same way and by taking this number to the next level of assessment of volatility in the return as compared to the average on a given year.

- **Rate of Risk: Standard Deviation of Periodic Returns**

  The expected return on a risky asset such stocks and bonds depend on systematic risk, such as the overall market movements, and unsystematic risk or portfolio-specific risk. We will examine later in detail how the unsystematic or idiosyncratic risk can be minimized or eliminated through diversification. First, it's important to examine the relationship between the rate of return and risk such as volatility, or the standard deviation of these returns. There are two methods of calculating the risk-return: historical analysis and scenario analysis.

Measuring Return and Quantifying Risk

Historical Analysis Method

HISTORICAL RETURN ANALYSIS

PORTOLIO A

Year	ROR	Deviation to return (X-Avg(X))	SQR Deviation
1	12.9%	3.0%	0.089%
2	31.3%	21.4%	4.571%
3	5.0%	-4.9%	0.242%
4	-2.0%	-11.9%	1.421%
5	2.0%	-7.9%	0.627%
6	22.0%	12.1%	1.459%
7	12.0%	2.1%	0.043%
8	12.0%	2.1%	0.043%
9	-5.0%	-14.9%	2.226%
10	9.0%	-0.9%	0.008%
Total	99.2%		10.730%

Observations= 10 n
Average = **9.920%** Total ROR / n
Variance = 1.192% Sqr Dev/ (n - 1)
Standard Dev.= **10.92%**

PORTOLIO B

Year	ROR	Deviation to return (X-Avg(X))	SQR Deviation
1	19.9%	10.0%	0.996%
2	15.0%	5.1%	0.258%
3	8.0%	-1.9%	0.037%
4	-11.0%	-20.9%	4.376%
5	7.0%	-2.9%	0.085%
6	14.0%	4.1%	0.166%
7	24.0%	14.1%	1.982%
8	20.0%	10.1%	1.016%
9	-5.4%	-15.3%	2.347%
10	7.7%	-2.2%	0.049%
Total	99.2%		11.314%

Observations= 10 n
Average = **9.920%** Total ROR / n
Variance = 1.257% Sqr Dev/ (n - 1)
Standard Dev.= **11.21%**

Figure 1.2

Measuring Return and Quantifying Risk

Scenario Analysis Method

STOCKS

SCENARIO PERFROMANCE ANALYSIS

Scenario (s)	Probability (p)	Stocks (s) ROR % (r_s)	$p * r_s$ %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-12.00	-3.00	-23.70	561.69	140.42
Normal (Sn)	45.0%	14.00	6.30	2.30	5.29	2.38
Boom (Sb)	30.0%	28.00	8.40	16.30	265.69	79.71
	100.0%		11.70 %		Variance=	222.51
					SD =	14.92 %

Figure 1.3

BONDS

SCENARIO PERFORMANCE ANALYSIS

Economic Scenario (s)	Probability (p)	Bonds (b) ROR % (r_b)	$p * r_b$ %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession	25.0%	14.00	3.50	9.75	95.06	23.77
Normal	45.0%	5.00	2.25	0.75	0.56	0.25
Boom	30.0%	-5.00	-1.50	-9.25	85.56	25.67
	100.0%		4.25 %		Variance=	49.69
					SD =	7.05 %

Figure 1.4

Return, Return Expectation, Risk and Allocation

Return:

Then the combined portfolio shown in figure 1.5 consisting of 60% stock and 40% bonds shows an expected combined return, variance, and standard deviation of 8.72%, 38.99% or .39x, and 6.24%, respectively. As expected, as we moved from the stock portfolio of 100% to a portfolio of 60% stock and 40% bonds, the return is calculated at 8.72% measured as

(Ws . Rs) + (Wb . Rb) = = (.60) (11.70%) + (.40) (4.25%) = 7.02% + 1.7% = 8.72%

Risk:

The Risk is measured by the amount of volatility needed to achieve the expected returns. The volatility is basically the variance and standard deviation of the historical rate change of the stocks during the 3 scenarios. The formulas areas follows:

Variance = $\sigma_P^2 = w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\rho$

Standard Deviation = $\sigma_P = \sqrt{(w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\ \rho)}$

Measuring Return and Quantifying Risk

Scenario Analysis Method

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation	Weights (W%)
Stocks (Ws) =	60%
Bonds (Wb) =	40%

$(Ws * rs) + (Wb * rb)$

Scenario (s)	Probability (p)	ROR % (rs)	p * rs %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-1.6	-0.40	-10.32	106.50	26.63
Normal (Sn)	45.0%	10.4	4.68	1.68	2.82	1.27
Boom (Sb)	30.0%	14.8	4.44	6.08	36.97	11.09
	100.0%		8.72 %		Variance=	38.99
					SD =	6.24 %

Figure 1.5

Number of Periods: Using Excel

- Type the formula
- Use a dialogue box
  - Click on Formulas > Financial > NPER.
  - Fill in the Rate, periodic payments (Pmt), present value (pv), and future value (Fv). Click OK.
- Type the function
  - =NPER(rate, pmt, pv, fv, type)

HPR = CF / investment

bought the stock at 100 sold it 120 = what's your return

(120-100)/100 = 20/100 = 20%

Bought 100, received a div $4 sold at 120 = HPR?

(120-100+4) / 100 = (20 + 4)/100 = 24/100 = .24 = 24%

INTERNAL RATE OF RETURN (IRR)

Dollar Weighted Return (Even Annual Payments)

	0	1	2	3	4
Net CF ($)	-100	5	5	5	105

EXCEL

	A	B	C	D	E	F
10	IRR	0	1	2	3	4
11	5.00%	-100	5	5	5	105

=IRR(B11:F11)

Dollar Weighted Return (Uneven Annual Payments

	0	1	2	3	4
Net CF ($)	-100	-9	-5	26	110

$$1 = + (-0.1 / (1+IRR) + (-0.5 / (1+ IRR)\ ^2 + (0.8 / (1+ IRR)^3) + (1.0 / (1+IRR)$$

EXCEL

	A	B	C	D	E	F
10	IRR	0	1	2	3	4
11	4.96%	-100	-9	-5	26	110

=IRR(B11:F11)

HISTORICAL RETURN ANALYSIS

PORTOLIO A	
Year	ROR
1	12.9%
2	31.3%
3	5.0%
4	-2.0%
5	2.0%
6	22.0%
7	12.0%
8	12.0%
9	-5.0%
10	9.0%
Total	99.2%

PORTOLIO B	
Year	ROR
1	19.9%
2	15.0%
3	8.0%
4	-11.0%
5	7.0%
6	14.0%
7	24.0%
8	20.0%
9	-5.4%
10	7.7%
Total	99.2%

SCENARIO analysis

		Stocks (s)				
Scenario (s)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-12.00				
Normal (Sn)	45.0%	14.00				
Boom (Sb)	30.0%	28.00				
	100.0%					