

## LECTURE 7

### Bond Prices, Yields and Portfolio Management (Chapters 10 & 11)

Money Terms:

- Amount
  - Face Value / Par Value (\$1,000)
  - Market Value quoted as a % of Face Value (priced at 98 or 98% of \$1,000)
- Coupon Payments / Coupon (Interest Rate)
  - ZERO COUPON PAYMENTS
  - Semi Annual Payments (interest payments)
  - Accrued Interest
    - $\text{Accr. Int.} = (\text{Annual Coupon} / 2) \times (\text{Days since last Coupon pmt} / \text{Days Separating Coupon Pmts})$

**Example:**

Par Value = \$1,000

Coupon = 4.25% therefore bond payment is \$42.50 per year in \$21.25 every 6 months

The Bid Price = 98:07 or 98 and 7/32 or 98.21875 % or MV = \$982.19

Bought it 32 days since the last coupon.

Accrued Interest pmt on the bond =  $\$21.25 \times (32/182) = \$7.47$ .

The purchase price =  $\$982.19 + \$7.47 = \$989.66$  (Invoice Price)

- External Ratings

		S&P	Moody's	
Risk Free	→	AAA	Aaa	}
NON- INVESTMENT GRADE	}	AA+	Aa1	
		AA	Aa2	
		AA-	Aa3	
		A+	A1	
		A	A2	
		A-	A3	
		BBB+	Baa1	
BBB	Baa2			
BBB-	Baa3			
NON- INVESTMENT GRADE	}	BB+	Ba1	
		BB	Ba2	
		BB-	Ba3	
		B+	B1	
		B	B2	
B-	B3			
DISTRESS	}	CCC+	Caa1	
		CCC	Caa2	
		CCC-	Caa3	
		CC	Ca	
C	C			
Defaulted	→	D	C	

Types of Bonds:

- Treasury Bonds (10-30yr) & Notes (10 yr)
- Corporate Bonds
  - Call Provisions – Call Price / Call Protection
  - Convertible Bonds – option to convert to common stock
    - Conversion Ratio – number of shares for each bond

**Example:**

Bond Par Value = \$1,000

Convertible ratio = 40 shares

At Current Stock = \$20 per share so the option to convert is unprofitable ( $\$20 \times 40 = \$800$  or *Market Conversion Value*)

At Current Stock = \$30 per share so the option to convert is profitable ( $\$30 \times 40 = \$1,200$  or *Market Conversion Value*)

- Conversion Premium is the excess of the bond price over its conversion value. If the bond were selling currently \$950, the stock is \$20 then its premium would be \$150 ( $\$950 - \$800$ )
  
- Puttable Bonds (option to the bond holders to put the bonds to the Issuer)
- Floating-rate Bonds – T + 2.0%
- PIK Bonds (Paid-in-Kind)
  
- Preferred Stock (Dividends – Waterfall ahead of the Common Stock )
- Other Domestic Bonds (Municipal, local governments, Tax exempt)
- International Bonds
  - Foreign Bonds
  - Eurobonds (Issued in the currency of one country but sold in other national market) – Eurodollar – dollar-denominated bonds sold outside the U.S.
  - Yankee Bonds (foreign bonds sold in the US)
  - Samurai Bonds (Yen-denominated bonds sold in Japan by non-Japanese issuers)
  - Bulldog Bonds (British Pound-denominated foreign bonds sold in the U.K.)

Bond Pricing

Bond Value = PV of Coupons + PV of Par Value at Maturity

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$$\text{Bond Value} = \sum (\text{Coupon Pmt} / (1 + r)^t) + (\text{Par Value} / (1 + r)^T)$$

Where,

Maturity Date = T – (using PV Factor tables)

Discount Rate = r

Years (t) – (using Annuity Factor tables)

$$\text{Coupon} * (1/r) [1 - (1 / ((1+r)^T))] + \text{Par Value} * (1 / ((1+r)^T))$$

Or

$$\text{Coupon} * \text{Annuity Factor} (r, T) + \text{Par Value} * \text{PV Factor} (r, T)$$

Table:

Example

Par Value: \$1,000

Coupon: 8.0% (4% or \$40 coupon payment every six months)

Maturity: 30 years (60 payments)

$$\text{Price} = \sum [\$40 / (1.04)^t] + [1000 / (1.04)^{60}]$$

$$\text{Price} = \$40 \times \text{Annual Factor} (4\%, 60) + \$1000 \times \text{PV Factor} (4\%, 60)$$

$$\text{Price} = \$ 904.94 + 95.06 = \$1,000$$

If the interest rates will rise to 10%

1	B	C	D	E	F	G	H
2	<b>BOND PRICING</b>						
3							
4	Par/Face Value	\$ 1,000.00		Semi-Annual Coupon =		4.00%	
5	Coupon % =	8.00%		Semi-Annual Payment =	\$	40.00	Every 6 months
6	Maturity/Term =	30 Yrs		Semi-Annual # Payments =		60 Pmts	
7							
8	Present Value of Coupon Pmts=			\$904.94	=PV(B4/2,G5,-G4)		
9	Present Value of Principal Pmt=			\$95.06	=PV(B4/2,G5,0,-B3,0)		
10	Total			<u>\$1,000.00</u>			

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11	11	B	C	D	E
12	Net Present Value		\$904.94	\$95.06	\$1,000.00
13			=NPV(\$B\$4/2,C16:C75)		
14					
15		<b>Long-Form</b>			
16		<b>Period</b>	<b>Coupon Payment</b>	<b>Principal Payment</b>	<b>Total Payment</b>
17		0			\$ (1,000.00)
18		1	\$ 40.00	\$ -	\$ 40.00
19		2	\$ 40.00	\$ -	\$ 40.00
20		3	\$ 40.00	\$ -	\$ 40.00
21		4	\$ 40.00	\$ -	\$ 40.00
22		5	\$ 40.00	\$ -	\$ 40.00
23		6	\$ 40.00	\$ -	\$ 40.00
24		7	\$ 40.00	\$ -	\$ 40.00
25		8	\$ 40.00	\$ -	\$ 40.00
26		9	\$ 40.00	\$ -	\$ 40.00
27		10	\$ 40.00	\$ -	\$ 40.00
28		11	\$ 40.00	\$ -	\$ 40.00
29		12	\$ 40.00	\$ -	\$ 40.00
30		13	\$ 40.00	\$ -	\$ 40.00
31		14	\$ 40.00	\$ -	\$ 40.00
32		15	\$ 40.00	\$ -	\$ 40.00
33		16	\$ 40.00	\$ -	\$ 40.00
34		17	\$ 40.00	\$ -	\$ 40.00
35		18	\$ 40.00	\$ -	\$ 40.00
36		19	\$ 40.00	\$ -	\$ 40.00
37		20	\$ 40.00	\$ -	\$ 40.00
38		21	\$ 40.00	\$ -	\$ 40.00
39		22	\$ 40.00	\$ -	\$ 40.00
40		23	\$ 40.00	\$ -	\$ 40.00
41		24	\$ 40.00	\$ -	\$ 40.00
42		25	\$ 40.00	\$ -	\$ 40.00
43		26	\$ 40.00	\$ -	\$ 40.00
44		27	\$ 40.00	\$ -	\$ 40.00
45		28	\$ 40.00	\$ -	\$ 40.00
46		29	\$ 40.00	\$ -	\$ 40.00
47		30	\$ 40.00	\$ -	\$ 40.00
48		31	\$ 40.00	\$ -	\$ 40.00
49		32	\$ 40.00	\$ -	\$ 40.00
50		33	\$ 40.00	\$ -	\$ 40.00
51		34	\$ 40.00	\$ -	\$ 40.00
52		35	\$ 40.00	\$ -	\$ 40.00
53		36	\$ 40.00	\$ -	\$ 40.00
54		37	\$ 40.00	\$ -	\$ 40.00
55		38	\$ 40.00	\$ -	\$ 40.00
56		39	\$ 40.00	\$ -	\$ 40.00
57		40	\$ 40.00	\$ -	\$ 40.00
58		41	\$ 40.00	\$ -	\$ 40.00
59		42	\$ 40.00	\$ -	\$ 40.00
60		43	\$ 40.00	\$ -	\$ 40.00
61		44	\$ 40.00	\$ -	\$ 40.00
62		45	\$ 40.00	\$ -	\$ 40.00
63		46	\$ 40.00	\$ -	\$ 40.00
64		47	\$ 40.00	\$ -	\$ 40.00
65		48	\$ 40.00	\$ -	\$ 40.00
66		49	\$ 40.00	\$ -	\$ 40.00
67		50	\$ 40.00	\$ -	\$ 40.00
68		51	\$ 40.00	\$ -	\$ 40.00
69		52	\$ 40.00	\$ -	\$ 40.00
70		53	\$ 40.00	\$ -	\$ 40.00
71		54	\$ 40.00	\$ -	\$ 40.00
72		55	\$ 40.00	\$ -	\$ 40.00
73		56	\$ 40.00	\$ -	\$ 40.00
74		57	\$ 40.00	\$ -	\$ 40.00
75		58	\$ 40.00	\$ -	\$ 40.00
76		59	\$ 40.00	\$ -	\$ 40.00
77		60	\$ 40.00	\$ 1,000.00	\$ 1,040.00
		<b>IRR =</b>	<b>4.00%</b>		

Valuing the Bonds

1	K	L	M	N	O	P
2	<b>VALUING BONDS</b>					
3						
4	Settlement Date=		1/15/2007			
5	Maturity Date=		1/15/2011			
6	Coupon Rate=		4.250%			
7	Yield to Maturity=		4.740%			
8	Redemption value %=		100			
9	Coupon Pmts per year=		2			
10						
11	Flat Price (% Par)		98.234	=PRICE(M4,M5,M6,M7,M8,M9)		
12	Day since last coupon=		0	=COUPDAYBS(M4,M5,2,1)		
13	Days in coupon period=		181	=COUPDAYS(M4,M5,2,1)		
14	Accrued Interest=		0	=(M12/M13)*M6*100/2		
15	Invoice Price=		98.234	=+M11+M14		
16						
17						
18	Settlement Date=		<b>2/15/2007</b>			
19	Maturity Date=		1/15/2011			
20	Coupon Rate=		4.250%			
21	Yield to Maturity=		4.740%			
22	Redemption value %=		100			
23	Coupon Pmts per year=		2			
24						
25	Flat Price (% Par)		98.264			
26	Day since last coupon=		<b>31</b>			
27	Days in coupon period=		181			
28	Accrued Interest=		0.36395028			
29	Invoice Price=		98.628			
30						

Yield to Maturity

81	B	C	D	E	F	G	H	
82	<b>YIELD TO MATURITY</b>							
83								
84	Settlement Date=		1/1/2000					
85	Maturity Date=		1/1/2010					
86	Coupon Rate=		8.000%					
87	Bond Pricing=		110					
88	Redemption Value=		100					
89	Coupon pmts per yr=		2					
90								
91	Yield to Maturity=		6.617% =YIELD(D84,D85,D86,D87,D88,D89)					

93	<b>Long-Form</b>			
94	Period	Coupon Payment	Principal Payment	Total Payment
95	0			\$ (1,100.00)
96	1	\$ 40.00	\$ -	\$ 40.00
97	2	\$ 40.00	\$ -	\$ 40.00
98	3	\$ 40.00	\$ -	\$ 40.00
99	4	\$ 40.00	\$ -	\$ 40.00
100	5	\$ 40.00	\$ -	\$ 40.00
101	6	\$ 40.00	\$ -	\$ 40.00
102	7	\$ 40.00	\$ -	\$ 40.00
103	8	\$ 40.00	\$ -	\$ 40.00
104	9	\$ 40.00	\$ -	\$ 40.00
105	10	\$ 40.00	\$ -	\$ 40.00
106	11	\$ 40.00	\$ -	\$ 40.00
107	12	\$ 40.00	\$ -	\$ 40.00
108	13	\$ 40.00	\$ -	\$ 40.00
109	14	\$ 40.00	\$ -	\$ 40.00
110	15	\$ 40.00	\$ -	\$ 40.00
111	16	\$ 40.00	\$ -	\$ 40.00
112	17	\$ 40.00	\$ -	\$ 40.00
113	18	\$ 40.00	\$ -	\$ 40.00
114	19	\$ 40.00	\$ -	\$ 40.00
115	20	\$ 40.00	\$ 1,000.00	\$ 1,040.00
116	<b>IRR =</b>			<b>3.3085%</b> 6.617%

	K	L	M	N	O	P	Q
82	<b>YIELD TO CALL Vs YIELD TO MATURITY</b>						
83							
84			<b>YTC</b>			<b>YTM</b>	
85	Settlement Date=		1/1/2000			1/1/2000	
86	Maturity Date=		1/1/2010			1/1/2030	
87	Coupon Rate=		8.00%			8.00%	
88	Coupon Pmt =	\$	40.00			\$ 40.00	
89	Number of semiannual		20 periods			60 periods	
90	Call Provision		110.00			100.00	
91	Final Payment		100.00			100.00	
92	Price		115.00			115.00	
93			_____			_____	
94	YIELD =		<u>6.6434%</u>			<u>6.8192%</u>	
95							
96							
97							

=YIELD(M85,M86,M87,M92,M91,2)

### Bond Portfolio (Chapter 11)

Interest Rate Sensitivity – Calculating Duration and Convexity

$$D_{Mac} = \frac{\sum_{t=1}^N \frac{CF_t}{(1+i)^t} t}{V_B}$$

$$C = \frac{1}{(1+i)^2} \left[ \frac{\sum_{t=1}^N \frac{CF_t}{(1+i)^t} (t^2 + t)}{V_B} \right]$$

**Duration:** is a measure of the sensitivity of the asset's price to [interest rate](#) movements. It broadly corresponds to the length of time before the asset is due to be repaid. This **duration** is equal to the ratio of the percentage reduction in the bond's price to the percentage increase in the [redemption yield](#) of the bond (or vice versa) (Lambda)

The standard definition of duration is [Macaulay duration](#), the PV-weighted time to receive each cash flow, defined as:

$$\text{Weighted Average } Wt = [cf / (1 + y)^t] / \text{Bond Price}$$

Y = yield to maturity

T=time

$$D = \sum t * Wt$$

100	K	L	M	N	O	P	Q
101	<b>Duration</b>						
102							
103	Int.Rate						P x 0
104	=	10%					
105			Time until	Payment	PV of Pmt	%	Duration
106			Payments		DR = 10%	Weight	
107	8% coupon bond		1	80	72.727	7.65%	0.0765
108			2	80	66.116	6.96%	0.1392
109			3	1080	811.420	85.39%	2.5617
110					950.263	100.00%	2.7774
111		Duration					
112	Zero						
113	Bond	will be 3 years					
114							
115							
116							

Duration is a key concept in bond portfolio management for at least 3 reasons:

1. It's a simple summary measure of the effective average maturity of the portfolio
2. It turns out to be an essential tool in immunizing portfolios from interest rate risk.
3. Duration is the measurement of the interest rate sensitivity of a bond portfolio.

Convexity

**Convexity** is a measure of the sensitivity of the [duration](#) of a [bond](#) to changes in [interest rates](#). There is an inverse relationship between convexity and sensitivity - in general, the



higher the convexity, the less sensitive the bond price is to interest rate shifts, the lower the convexity, the more sensitive it is.

Duration is a [linear](#) measure or 1st derivative of how the price of a bond changes in response to interest rate changes. As interest rates change, the price is not likely to change linearly, but instead it would change over some curved [function](#) of interest rates. The more curved the price function of the bond is, the more inaccurate duration is as a measure of the interest rate sensitivity.

Convexity is a measure of the curvature or 2nd derivative of how the price of a bond varies with interest rate, i.e. how the duration of a bond changes as the interest rate changes.

$$\Delta P / P = - D \times \Delta y$$

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100	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
101	<b>MACAULAY DURATION AND CONVEXITY</b>													
102	<b>Sensitivity to interest rate movements</b>											<b>IRR= 10.0000%</b>		
103														
104	Bnd Price	\$875.38					<b>If Yield Changes By</b>	<b>1.00%</b>						
105	Face Value	1.00					Bnd Price Will Change By	-54.63	-6.24%	=+R105/M104				
106	Coupon Rate	8.00%												
107	Life in Years	10					Modified Duration Predicts	-57.03		=(M104+PV((M108+R104)/M109,M107*M109,M106*M105/M109,M105))				
108	Yield	10.00%					Convexity Adjustment	2.47		=-0.5*M113*R104^2*M104				
109	Frequency	2					Total Predicted Change	-54.56		=+R107+R108				
110														
111	<b>Macauly Duration</b>	<b>6.84</b>					Actual New Price	\$820.74		=PV((M108+Q104)/M109,M107*M109,M106*M105/M109,M105))				
112	Modified Duration	6.51					Predicted New Price	\$820.82		=+M104+R109				
113	Convexity	56.49					Difference	\$0.08		=+R112-R111				
114														
115	<b>Period</b>	<b>Cash Flow</b>	<b>PV Cash Flow</b>	<b>Weighted</b>	<b>Duration Calc Method1</b>	<b>Duration Calc Method2</b>	<b>PV of pv(CF)</b>	<b>Factor</b>	<b>years</b>	<b>Convexity Calc</b>				
116	0	(\$875.38)												
117	1	40.00	38.10	4.352%	0.04352	38.10	34.554	2.000		69.11				
118	2	40.00	36.28	4.148%	0.08289	72.56	32.908	6.000		197.45				
119	3	40.00	34.55	3.947%	0.11842	103.66	31.341	12.000		376.09				
120	4	40.00	32.91	3.759%	0.15037	131.63	29.849	20.000		596.97				
121	5	40.00	31.34	3.580%	0.17901	156.71	28.427	30.000		852.82				
122	6	40.00	29.85	3.410%	0.20459	179.09	27.074	42.000		1,137.09				
123	7	40.00	28.43	3.247%	0.22732	198.99	25.784	56.000		1,443.92				
124	8	40.00	27.07	3.093%	0.24742	216.59	24.557	72.000		1,768.07				
125	9	40.00	25.78	2.948%	0.26510	232.06	23.387	90.000		2,104.85				
126	10	40.00	24.56	2.809%	0.28052	245.57	22.273	110.000		2,450.08				
127	11	40.00	23.39	2.672%	0.29388	257.26	21.213	132.000		2,800.10				
128	12	40.00	22.27	2.544%	0.30533	267.28	20.203	156.000		3,151.62				
129	13	40.00	21.21	2.423%	0.31503	275.77	19.241	182.000		3,501.80				
130	14	40.00	20.20	2.308%	0.32310	282.84	18.324	210.000		3,848.14				
131	15	40.00	19.24	2.198%	0.32970	288.61	17.452	240.000		4,188.45				
132	16	40.00	18.32	2.093%	0.33493	293.19	16.621	272.000		4,520.86				
133	17	40.00	17.45	1.994%	0.33892	296.68	15.829	306.000		4,843.78				
134	18	40.00	16.62	1.899%	0.34177	299.17	15.076	342.000		5,155.85				
135	19	40.00	15.83	1.808%	0.34357	300.76	14.358	380.000		5,455.92				
136	20	1,040.00	391.97	44.777%	8.95533	7,839.30	355.524	420.000		149,320.02				
137	<b>Total</b>	<b>\$875.38</b>		100%	13.68074	11,975.81	793.99366			197,783.01				
138	<b>PRICE</b>				<b>DURATION</b>	<b>6.84037</b>				<b>CONVEXITY</b>	<b>56.49</b>			
139														
140	K	L	M	N	O	P	Q	R	S	T	U	V	W	X

## Bond Terminology

### Accrued Interest

Accrued interest is the interest that has been earned, but not yet been paid by the bond issuer, since the last coupon payment. Note that interest accrues equally on every day during the period. That is, it does not compound. So, halfway through the period, you will have accrued exactly one-half of the period's interest payment. It works the same way for any other fraction of a payment period.

### Banker's Year

A banker's year is 12 months, each of which contains 30 days. Therefore, there are 360 (not 365) days in a banker's year. This is a convention that goes back to the days when "calculator" and "computer" were job descriptions instead of electronic devices. Using 360 days for a year made calculations easier to do. This convention is still used today in some calculations such as the Bank Discount Rate that is used for discount (money market) securities.

### Bond

A bond is a debt instrument, usually tradable, that represents a debt owed by the issuer to the owner of the bond. Most commonly, bonds are promises to pay a fixed rate of interest for a number of years, and then to repay the principal on the maturity date. In the U.S. bonds typically pay interest every six months (semi-annually), though other payment frequencies are possible. Bonds are issued by corporations, banks, state and local governments (municipal bonds), and the federal government (Treasury Notes and Bonds).

### Call Date

Some bonds have a provision in the indenture that allows for early, forced, redemption of the bond, often at a premium to its face value. Bonds that have such a feature usually have a series of such dates (typically once per year) at which they can be called. This series of dates is referred to as the call schedule.

### Call Premium

The extra amount that is paid by a bond issuer if the bond is called before the maturity date. This is a sweetener that is used to make callable bonds attractive to investors, who would otherwise prefer to own non-callable bonds.

### Clean Price

The "clean price" is the price of the bond excluding the accrued interest. This is also known as the quoted price.

### Coupon Payment

This is the actual dollar amount that is paid by the issuer to the bondholders at each coupon date. It is calculated by multiplying the [coupon rate](#) by the [face value](#) of the bond and then dividing by the number of payments per year.

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## **Coupon Payment Date**

The specified dates (typically two per year) on which interest payments are made.

## **Coupon Rate**

The stated rate of interest on the bond. This is the annual interest rate that will be paid by the issuer to the owners of the bonds. This rate is typically fixed for the life of the bond, though variable rate bonds do exist. The term is derived from the fact that, in times past, bond certificates had coupons attached. The coupons were redeemed for cash payments.

## **Current Yield**

A measure of the income provided by the bond. The current yield is simply the annual interest payment divided by the current market price of the bond. The current yield ignores the potential for capital gains or losses and is therefore not a complete measure of the bond's rate of return.

## **Day-count Basis**

A method of counting the number of days between two dates. There are several methods, each of which makes different assumptions about how to count. **30/360** (a banker's year) assumes that each month has 30 days and that there are 360 days in a year. **Actual/360** counts the actual number of days, but assumes that there are 360 days in a year. **Actual/Actual** counts the actual number of days in each month, and the actual number of days in a year. In Excel bond functions, 0 signifies 30/360, 1 specifies actual/actual, 2 is actual/360, 3 is actual/365 (which ignores leap days), and 4 represents the European 30/360 methodology.

## **Dirty Price**

The "dirty price" is the total price of the bond, including accrued interest. This is the amount that you would actually pay (or receive) if you purchase (or sell) the bond.

## **Face Value**

The principal of a bond is the notional amount of the loan. It is also called the **principal** or **par** value of the bond, and represents the amount that will be repaid when the bond matures.

## **Indenture**

The legal contract between a bond issuer and the bondholders. The indenture covers such things as the original term to maturity, the interest rate, interest payment dates, protective covenants, collateral pledged (if any), and so on.

## **Maturity Date**

The date on which the bond ceases to earn interest. On this date, the last interest payment will be made, and the face value of the bond will be repaid. This is also sometimes known as the **redemption date**.

## **Redemption Value**

This is typically the same as the face value of a bond. However, for a callable bond, it is the face value plus the call premium. In other words, this is the entire amount that will be received when the bond is redeemed by the issuer.

## **Settlement Date**

The date on which ownership of a security actually changes hands. Typically, this is several days after the trade date. In the US markets, the settlement date is usually 3 trading days after the trade date (this is known as T+3). For bonds, a purchaser begins to accrue interest on the settlement date.

## **Term to Maturity**

The amount of time until the bond stops paying interest and the principal is repaid.

## **Yield to Call**

Same as yield to maturity, except that we assume that the bond will be called at the next call date. Also known as yield to first call. Frequently, the yield to all call dates is calculated, and then we can find the worst-case, which is known as the yield to worst.

## **Yield to Maturity**

The yield to maturity (YTM) of a bond is the compound average annual expected rate of return if the bond is purchased at its current market price and held to maturity. Implicit in the calculation of the YTM is the assumption that the interest payments are reinvested for the life of the bond at the same yield. The YTM is the internal rate of return (IRR) of the bond.

## **Yield to Worst**

The lowest of all possible yields for the bond. It is calculated by determining the minimum of the yield to maturity or any of the various yields to call date