LECTURE 6

Review – Option Valuations - Summary

- Option values may be viewed as the sum of intrinsic value plus time or "volatility" value. The volatility value is the right to choose not to exercise if the stock price moves against the holder. Thus, option holders cannot lose more the cost of the option regardless of stock price performance.
- Call options are more valuable when the exercise price is lower, when the stock price is higher, when the interest rate is higher, when the interest rate is higher, when the time to expiration is greater, when the stock's volatility is greater and when dividends are lower.
- Options may be priced relative to the underlying stock price using a simple two-period, two-stage pricing model (Binomial Model). As the number of periods increases, the model can approximate more realistic stock price distributions.
- The Black-Scholes Model Pricing -

$$\mathbf{C = S\,e^{-\delta T}\,N(d1) - X\,e^{-iT}\,N(d2)}$$

$$\mathbf{P = X\,e^{-iT}\,(1 - N(d2)) - S\,e^{-\delta T}\,(1 - N(d1))}$$

Volatility is the question on the B/S –which assumes constant SD throughout the exercise period - The time series of implied volatility

THE PUT – CALL PARITY RELATIONSHIP

- Put prices can be derived simply from the prices of call
- European Put or Call options are linked together in an equation known as the Put-Call parity relationship

	St <= X	**St > X**
Payoff of Call Held	0	St - X
Payoff of Put Written	-(X – St)	0
Total	St – X	St – X

PV (x) = X e^{-rt}

The option has a payoff identical to that of the leveraged equity position; the costs of establishing them must be equal

- C – P Cost of Call purchased = Premium received from Put written
- The leverage Equity position requires a net cash outlay of S – X e^{-rt} the Cost of the stock less the process from borrowing
- C – P = S – X e^{-rt} PUT-CALL Parity Relationship - proper relationship between Call and Put

S = $110
C = $14 for 6 months with X = $105
P = $5 for 6 months with X=$105
rf = 5.0% (continuously compounding at e)

Assumptions:

C – P = S – X e^{-rT} ?????
14 – 5 = 110 – 105 .$e^{-0.5 \times 0.5}$
9 = 7.59

This a violation of parity…. Indicates mispricing and leads to Arbitrage Opportunity

You can buy relatively cheap portfolio (buy the stock plus borrowing position represented on the right side of the equation and .sell the expensive portfolio

<u>ARBITRAGE STRATEGY</u> given the mispricing– In six months the stock will be worth Sr, so you borrow PV of X ($105) and pay back the loan with interest resulting in cash outflow of $105

Sr – 105 writing the call if Sr exceeds 105
Purchase Puts will pay 105 – Sr if the stock is below the $105

	Strategy	Immediate CF	CF if Sr < 105	CF if Sr > 105
1	Buy Stock	-110.00	Sr	Sr
2	Borrow Xe^{-iT} = \$102.41	+102.41	-105	-105
3	Sell Call	14.00	0	-(Sr – 105)
4	Buy Call	-5.00	105 – Sr	0
		1,41	0	0

Whish is the difference of between 9.00 and 7.59 – riskless return

This applies if No dividends and under the European option

If Dividend then

P = C – S + PV (X) + PV (Dividend) …. Representing that the Dividend (δ) is paid during the life of the option.

<u>Example</u>

Using the IBM example – today is February 6

X = \$100 (March calls)
T = 42 days
C = \$2.80
P = \$6.47
S = 96.14
I = 2.0%
δ = 0

P = C – S + PV (X) + PV (Dividend) or P = C + PV (X) – S + PV (δ)

???? $6.47 = 2.80 + 100 / (1+0.02)^{42/365} - 96.14 + 0$

6.47 = 6.63 is not that valuable to go after the repricing arbitrage

PUT OPTION VALUATION

$$P = X e^{-iT} (1- N (d2)) - S e^{-\delta T} (1 - N (d1))$$

Using the data from previous example

$P = 95.\ e^{-10x.25}\ (1 - 0.05714) - 100\ (1 - 0.6664)$

P = 6.35

PUT-CALL Parity

P = C + PV (X) – So + PV (Div)

\P = $13.70 + 95\ .\ e^{-10\ X\ 0.25} - 100 + 0$
Hedge Ratios & the B/S format

The Hedge ratio is commonly called the **Option Delta**. Is the change in the price of call option for $1 increased in the stock price

This is the slope of value function evaluated at the current stock price

For Example

Slope of the curve at S = $120 equals .60. As the stock increases by $1, the option increase on 0.60

For every Call Option Written, .60 shares of stock would be needed to hedge the Investment portfolio.

For example, if one writes 10 options and holds 6 shares of stock,

H = .60 ……… a $1 increase in stock will result $6 gain ($1x 6 shares) and with the loss of $6 on 10 options written (10 x $0.60)

- The Hedge Ratio for a Call is N (d1),
- with the hedge ratio for a Put [N (d1) – 1]
- N (d) is the area under standard deviation (normal)
- Therefore, the Call option Hedge Ratio must be positive and less than 1.0
- And the Put option Hedge Ratio is negative and less than 1.0

Example:

2 Portfolios

Portfolio	A	B
BUY	750 IBM Calls 200 Shares of IBM	800 shares of IBM

Which portfolio has a greater dollar exposure to IBM price movement?

Using the Hedge ratio you could answer that question:

Each Option change in value by H dollars for each $1 change in stock price

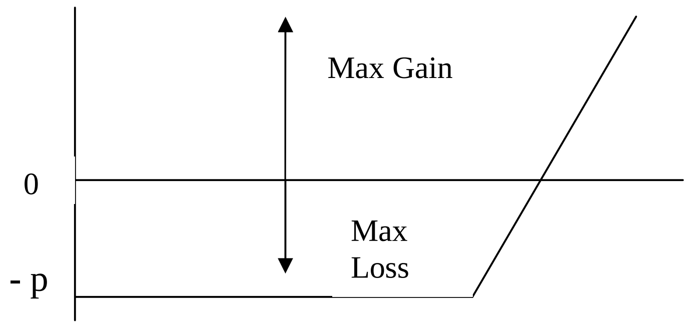
If H = 0.6, then 750 options = equivalent 450 shares (0.6 x 750)

Portfolio A = 450 equivalent + 200 shares which is less than Portfolio B with 800 shares

PORTFOLIO INSURANCE (PROTECTIVE PUT STRATEGY)

MAX LOSS:
At the money (X = S) the maximum loss than can be realized is the cost of the Put

MAX GAIN
Unlimited (sale of the stock)



Desired horizon of the Insurance Program must match the maturity of a traded option in order to establish the appropriate put positions

Most options don't go over 1 year – There is an Index options LEAPS (Long Term Equity AnticiPation Securities)

Synthetic Protective Puts – gives you a hedge mechanism without buying an option.

Example– Chasing the Deltas – Synthetic Protective Puts

Portfolio of $100 million
At the money Put Option on the portfolio with a Hedge ratio or Delta = 0.06 which means the option value swings $0.60 for every $1 change in the opposite direction

Portfolio goes DOWN by 2%

The profit of the hypothetical protective Put position (if the put existed) will be as follows:

Loss on stock	2.% x 100 =	$2.0 million
Gain on Put	.6 x 2.00 =	$1.2 million
Gain (loss)		$0.8 loss

We created the synthetic option position by selling a proportion of shares equal to the put option or Delta… ie. Sells 60% of the shares and the proceeds are placed in rf (risk free rate) T-Bills –

DYNAMIC HEDGING – Constant updating of **hedge positions as market conditions change**

1. A SIMPLE EXAMPLE OF DYNAMIC HEDGING

To start off, consider the following example, which we have adapted from Hull (1997): A financial institution has sold a European call option for $300,000. The call is written on 100,000 shares of a non-dividend paying stock with
the following parameters:

Current stock price = $49
Strike price = X = $50
Stock volatility = 20%
Risk-free interest rate r = 5%.
Option time to maturity T = 20 weeks
Stock expected return = 13%

The Black-Scholes price of this option is slightly over $240,000.

It follows that the financial institution has earned approximately $60,000 from writing the call. However, unless the financial institution *hedges* its obligation in order to offset the effects of price changes in the future, it could stand to lose much money at the call's expiration. For example, if the price of the stock at the option's expiration date is $60, and if the call is not hedged, then the financial institution will lose $1,000,000 (=100,000*(60-50)) at the option's expiration.

<u>Strategies include:</u>

1. Stop-Los Strategy
2. Delta Hedging