LECTURE 4

Chapter 6:

EFFICIENT DIVERSIFICATION
How investors can construct the best possible risky portfolio – efficient Diversification

“Diversification reduces the variability of portfolio returns”

DIVERSIFICATION AND PORTFOLIO RISK

From one stock to two stocks to three stocks….. sensitivity to external factors (i.e. oil, non-oil stocks) – But even extensive diversification cannot eliminate risk – MARKET RISK

- Other Names for Market risk: Systematic risk, non-diversifiable risk
- The Risk that can be eliminated by diversification is called:
    - Unique Risk
    - Firm-specific risk
    - Non-systematic risk
    - Diversifiable risk

ASSET ALLOCATION

Asset allocation between 2 risky assets

COVARIANCE AND CORRELATION

Relationship between the return of two assets

1. Tandem
2. Opposition

Depends on the Correlation between the two returns

Use the Economic Scenarios between two asset classes (Stocks and Bonds)

PERFORMANCE SCENARIOS

Stocks (s)

Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	30.0%	-11.00	-3.30	-21.00	441.00	132.30
Normal (Sn)	40.0%	13.00	5.20	3.00	9.00	3.60
Boom (Sb)	30.0%	27.00	8.10	17.00	289.00	86.70
	100.0%		10.00 %		Variance=	222.60
					SD =	14.92 %

Bonds (b)

Scenario (S)	ROR % (r_b)	p * r_b %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	16.00	4.80	10.00	100.00	30.00
Normal (Sn)	6.00	2.40	0.00	0.00	0.00
Boom (Sb)	-4.00	-1.20	-10.00	100.00	30.00
		6.00 %		Variance=	60.00
				SD =	7.75 %

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation

Stocks (As) =	60%
Bonds (Ab) =	40%

(As * r_s) + (Ab * r_b)

Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	30.0%	-0.2	-0.06	-8.60	73.96	22.19
Normal (Sn)	40.0%	10.2	4.08	1.80	3.24	1.30
Boom (Sb)	30.0%	14.6	4.38	6.20	38.44	11.53
	100.0%		8.40 %		Variance=	35.02
					SD =	5.92 %

COVARIANCE & CORRELATION

Scenario (S)	Probability (p)	Stocks (Deviation from the mean)	Bonds (Deviation from the mean)	Ds * Db	Covariance [p * (Ds*Db)
Recession (Sr)	30.0%	-21.00	10.00	-210.00	-63.00
Normal (Sn)	40.0%	3.00	0.00	0.00	0.00
Boom (Sb)	30.0%	17.00	-10.00	-170.00	-51.00
	100.0%			Covariance=	-114.00
				Correlation Coefficient =	-0.99

The Covariance is calculated in a manner similar to the Variance. Instead of measuring the typical difference of an asset return from its expected value.

Instead measure the extent to which the variation in the returns of the two assets tend to reinforce or offset each other

COVERIANCE

$$\text{Cov (rs.rb)} = \sum \text{p (i) [rs (i) – avg rs] [rb (i) – Avg rb]}$$

Rs = return on the stock
Rb = return on the bond
P (i) = expected portfolio return

CORRELATION COEFFICIENT

$$P_{sb} = \text{Cov (rs,rb)} / \sigma s \cdot \sigma b$$

Psb = portfolio of Stocks and bonds
σs = Standard Deviation of s
σb = Standard Deviation of b

THE 3 RULES OF TWO-RISKY ASSET PORTFOLIOS

Rule 1: ROR of the portfolio is weighted average of the returns

rp = Wb*rb + Ws*rs

Rule 2: Expected ROR or the portfolio

E (rp) = Wb*E (rb) + Ws*E (rs)

Rule 3: Variance of ROR or two-risky asset portfolio.

σp^2= (Wb*σb)^2 + (Ws*σs)^2 + **2 (Wb*σb) (Ws*σs)* Pbs**

Pbs is the correlation between the return on stock and bonds

Example: 100% Bonds, then decide to shift to 50% of bonds and 50% of stock

Input Data:

E(rb) = 6.0%
E(rs) = 10%
σb= 12%
σs= 25%
Pbs = 0
Wb=0.5
Ws=0.5

σp^2=(0.5*12)^2 + (0.5*25)^2 + 2(0.5*12)(0.5*25)*0

σp = SqRt of 192.25 = 13.87%

If we averaged the 2 standard deviations of each asset class we will have incorrectly predicted an increase in the portfolio's SD (25 + 12)/2 = 18.5% showing an increase of 6.5% when moving from all bond portfolio to half/half bond/stock. The actuality is that the SD movement is much lower to 13.87% (as is calculated above) or 1.87% from all bond portfolio SD of 12.0% - **SO THE GAIN OF DIVERSIFICATION CAN BE SEEN AS FULL 6.50 – 1.87 = 4.62%.**

*If weights 0.75 and 0.25 then (0.75*6) + (0.25*10) =* ***7.0% expected returns***

*Variance = (0.75*12) ^2 + (0.25*25)^2 + 2(0.75*12) (0.25*25) *0*

SqRt of 120 = ***10.96%***

Check page 159 – Graph and Table at rs=10, rb=6, σs=25, σb=12 at different weights

Parameters

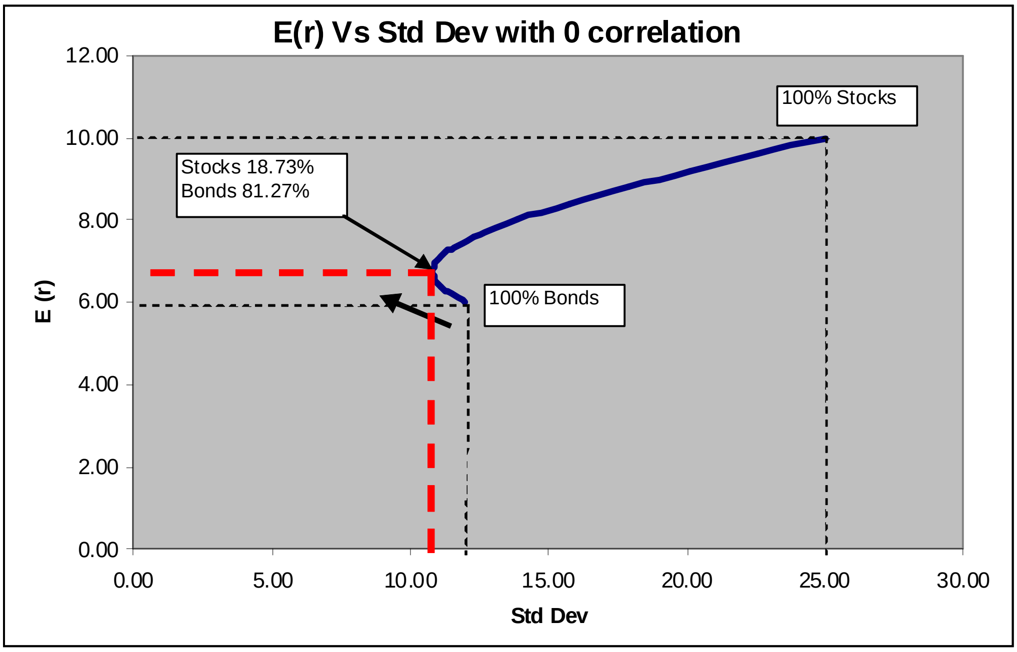
E (rs) =	**10**
E (rb) =	**6**
σs =	**25**
σb=	**12**
Psb =	**0**

Portfolio Weights		Exp Return	Std Dev.
Ws	**Wb**	**E(rp) %**	**σp %**
0.0	1.0	6.00	12.00
0.1	0.9	6.40	11.09
0.2	0.8	6.80	10.82
0.3	0.7	7.20	11.26
0.4	0.6	7.60	12.32
0.5	0.5	8.00	13.87
0.6	0.4	8.40	15.75
0.7	0.3	8.80	17.87
0.8	0.2	9.20	20.14
0.9	0.1	9.60	22.53
1.0	0.0	10.00	25.00

Minimum Variance

Stocks	18.7256%
Bonds	81.2744%

Ws=(σb^2 - σb σs p) / (σs^2 + σb^2 - 2*σb σs p)
Wb = 1 - Ws



The Mean – Variance Criterion

Investors Desire portfolios to lie to the Northwest (Graph) – with higher return and lower Standard Deviation (Risk)

Let's assume Portfolio A is said to dominate portfolio B if all investors prefer A over B. This will be the case that has the highest Return and lost Variance

$E(r_A) \geq E(r_B)$ and $\sigma_A \leq \sigma_B$

If we graph the relationship PA will be to the Northwest of PB

WHAT ARE THE IMPLICATIONS OF PERFECT POSITIVE CORRELATION BETWEEN BONDS & STOCKS??

Let's say the correlation is 1 or Pbs = 1 (so far we used 0 correlation)

Pbs = 1

σp^2 = Wb^2 σb ^2 + Ws^2 σs^2 + 2 Wb*σb Ws*σs * 1 = Wb*σb + Ws*σs)

so if Pb = 1 then σp = Wb*σb + Ws*σs

we learned if

Pb = 0 then σp = SqRt of (Wb*σb)^2+ (Ws*σs)^2

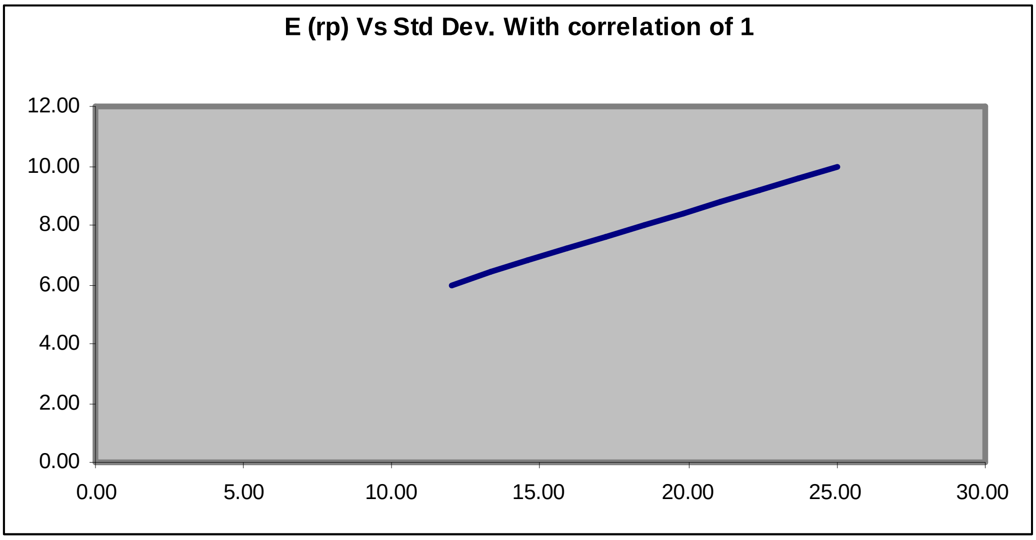
Example we were using (σs = 25, σb = 12)

σp= (.50 * 12) + (.50 * 25) = 18.75% …. If Pbs = 1, straight average – No gain for diversification, where Pbs = 0 we calculated previously that the σp = 13.87%

Graph of Pbs = 1 and Pbs = 0 and in between

With Correlation = 1
Psb = 1

Portfolio Weights		**Std Dev.**	**Exp Return**
Ws	**Wb**	**σp %**	**E(rp) %**
0.0	1.0	12.00	6.00
0.1	0.9	13.30	6.40
0.2	0.8	14.60	6.80
0.3	0.7	15.90	7.20
0.4	0.6	17.20	7.60
0.5	0.5	18.50	8.00
0.6	0.4	19.80	8.40
0.7	0.3	21.10	8.80
0.8	0.2	22.40	9.20
0.9	0.1	23.70	9.60
1.0	0.0	25.00	10.00



Use Extreme Example where **Pbs = -1**

σp^2 = (Wb*σb – Ws*σs)^2

or σp = ABS Wb*σb – Ws*σs

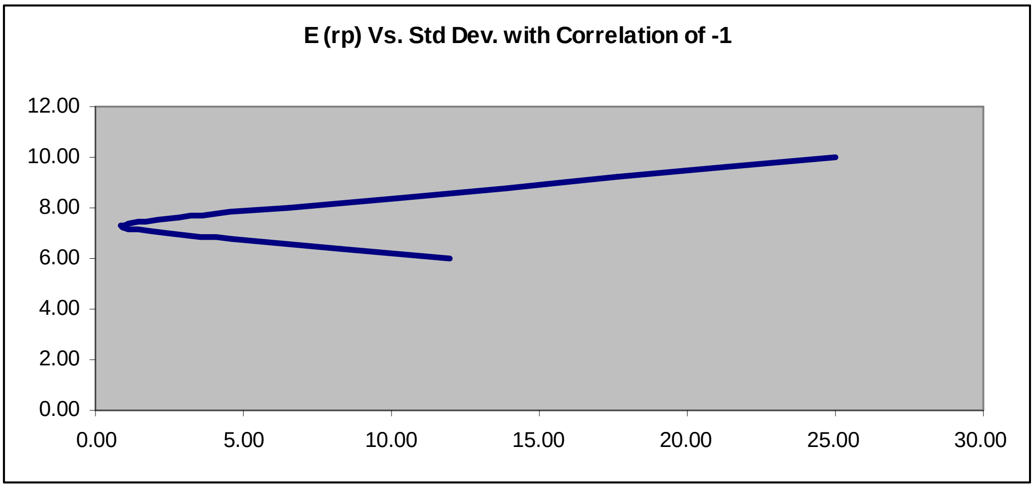
(using ABS or absolute because there is no negative standard deviation)

using our example = .50*12 - .50*25 = Abs 6.5%

With Correlation = -1

Psb = -1

Portfolio Weights		Std Dev.	Exp Return
Ws	**Wb**	**σp %**	**E(rp) %**
0.0	1.0	12.00	6.00
0.1	0.9	8.30	6.40
0.2	0.8	4.60	6.80
0.3	0.7	0.90	7.20
0.4	0.6	2.80	7.60
0.5	0.5	6.50	8.00
0.6	0.4	10.20	8.40
0.7	0.3	13.90	8.80
0.8	0.2	17.60	9.20
0.9	0.1	21.30	9.60
1.0	0.0	25.00	10.00



THE OPTIMAL RISKY PORTFOLIO W/ A RISK-FREE ASSET

Let's add Risk Free in our portfolio (bringing what we discussed before regarding CAL line)

Historical Correlation between Bonds and Stocks is 0.20

T-Bills = 5.0% (risk free)

GRAPH introducing the CAL in our previous Graph of Bonds and Stock

Using the minimum (point A) on a .20 correlation between bonds and stock. We were given the minimum weights at Wb= 87.06% and Ws = 12.94% so PA expects to return 6.52% and σA is 11.54% calculated as follows:

rA = (.8706 * 6) + (.1294 * 10) = 6.52

σA=(.8706 * 12) ^2 + (.1294 * 25) ^2 = 11.54%

Sharpe Ratio is SA = (E (rA) – rf) / σA = (6.52 – 5) / 11.54 = 0.13

Now consider the CAL uses portfolio B instead of A. Portfolio B consists of 80% Bonds and 20% Stock, then rbs = 6.80%, σbs = 11.68% then,

SB = (6.80 – 5) / 11.68 = 0.15

SB – SA = 0.02

This implies that portfolio B provides 2 extra basis points (0.02%) of expected return for every percentage point (1.0%) increased in Standard Deviation (Risk)
The higher Sharpe Ratio of B means that its capital allocation line (CAL) it's steeper than A, therefore, CAL(B) plots above CAL(A).
In other words, combination of portfolio B and the risk-free asset provide a higher expected return for any level of risk (SD) than combination of portfolio A and the risk free risk.

GOAL = **CAL NEED TO REACH TANGENCY (GRAPH) FOR OPTICAL RISKY PORTFOLIO**

Graph 6.6, page 166

Solution for maximizing of the Sharpe Ratio:

Wb = [(E(rb) – rf).σs^2 – (E9rs) – rf)*σb*σs*Pbs] / [(E (rb) – rf) σs^2 + (E (rs) – rf)*σb^2 – rf + E (rs) – rf*σb*σs*Pbs

Ws = 1 - Wb

BUILDING A PORTFOLIO WITH RISK FREE, STOCK, AND BONDS

Assume we want to invest 45% of our portfolio in Risk Free assets = 55% is in a risky portfolio between bonds (50%) and stocks (50%),

We find the CAL with our optimal portfolio (o) in a slope – Lets say:

Pro = 8.68% and σ0=17.97%, Wb = 32.99% and Ws = 67.01% from the long formula above.

So = 8.68 – 5 / 17.97 = 0.20

E(rc) = 5 + 0.55 *(8.68 – 5) = 7.02%
σc = 0.55 * 17.97 = 9.88%

Wrf = 45%
Wb = 0.3299 * .55 = 18.14%
Ws = 0.6701 * .55 = 36.86%

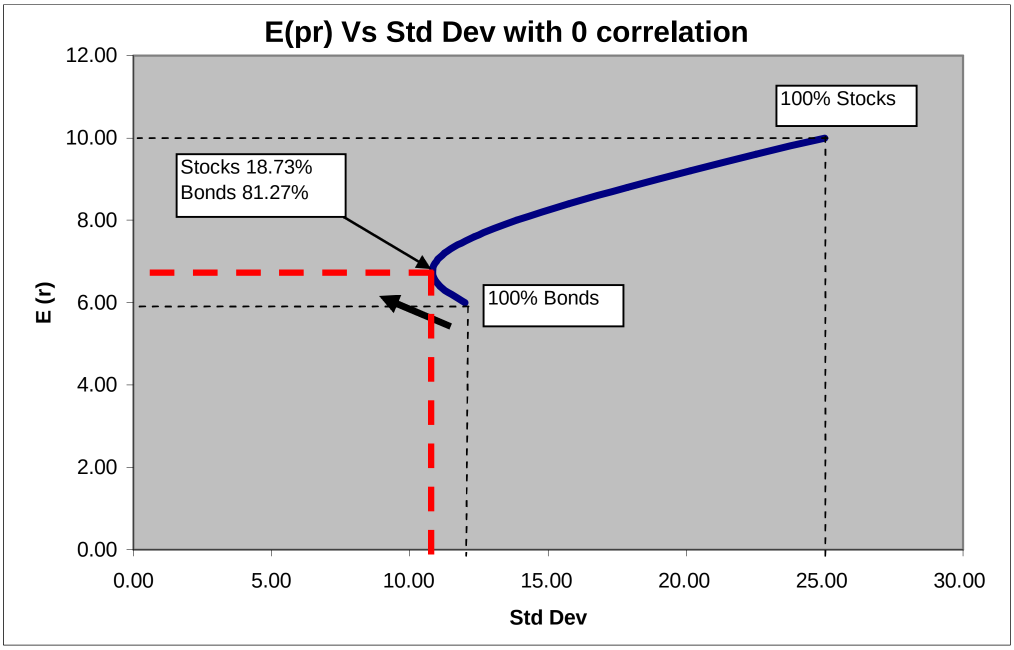
REVIEW – CHAPTER 6

THE EFFICIENT FRONTIER OF RISKY ASSETS

3 STEPS:

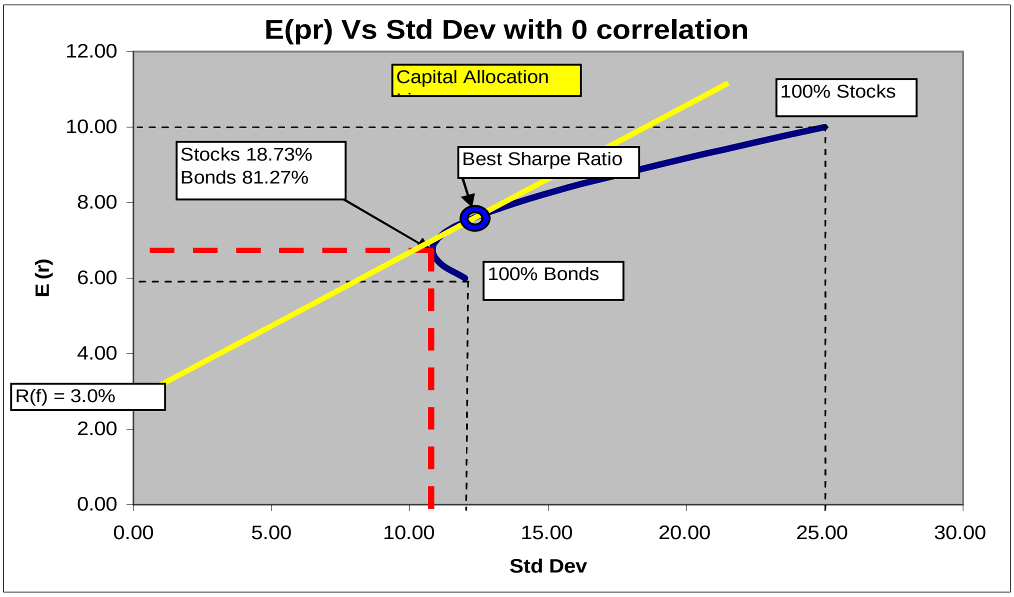
STEP 1:
Identify the best possible or most efficient risk-return combination available from the universe of risky assets (Plot them on Return/Standard Deviation Graph)

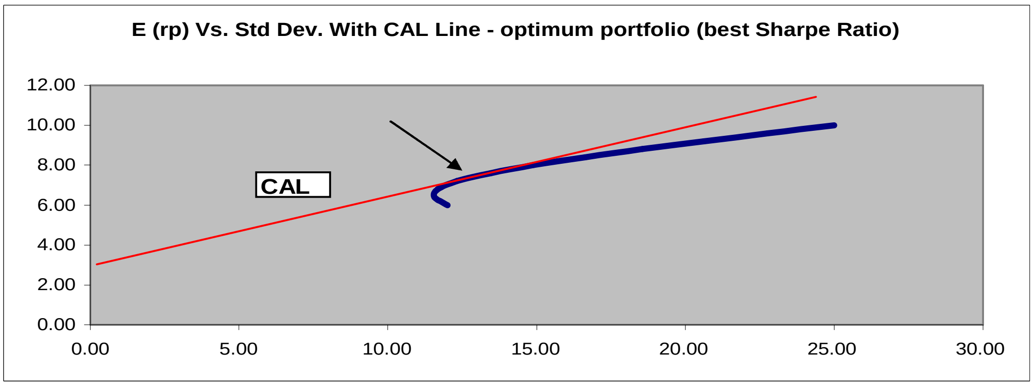
> *Expected Return – SD combination for any individual asset end-up inside the efficient frontier, because single-asset portfolios are inefficient (are not efficiently diversified)*



STEP 2:
Determine the optimal portfolio of risky assets by finding the portfolio that supports the steepest CAL (Risky free return introduced)

> *Risky free assets – using the current Risk Free Rate, we search for CAL with the highest Sharpe Ratio*





STEP 3:
Choose an appropriate complete portfolio based on the investors risk appetite (risk aversion) by mixing the Rf Asset with the optimal risky portfolio.

> *Choose the appropriate optimal risky portfolio (o) above T-bills – Separation Property step - RISK AVERSE comes to play in this step – when selected the desire point of the CAL. More risk averse clients will invest in the risk-free asset and less in the optimal risky portfolio O.*