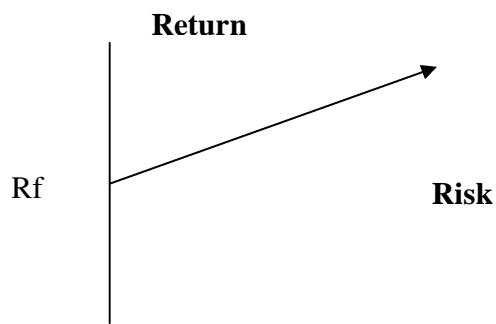


## LECTURE 3

### Chapter 5



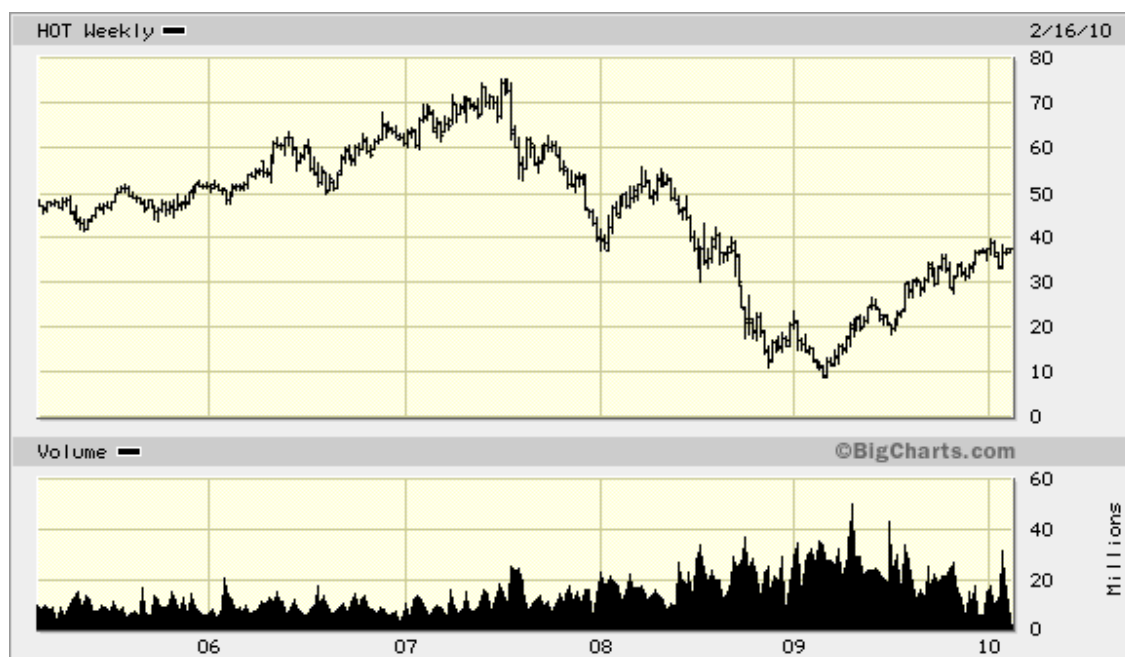
### RISK RETURN –

- Traditionally, when you define return you refer to a bank savings account (risk free) plus a risky portfolio of US stocks. Today, investors have access to a variety of asset classes and financial engineered investments
- The Book “The Quants” by Scott Peterson – financial engineering – achieving the ALPHA.

$$\text{HPR} = (\text{Ending Price} - \text{Beg. Price} + \text{Div}) / \text{Beg. Price}$$

Example:

Current Price = \$100, expected price to increase to \$110 in a year. Within the year you are expected to receive \$4 dividend, therefore the  $\text{HPR} = (110 - 100 + 4) / \$100 = 14\%$



Starwood Hotels  
5-yr stock prices

Measuring Return over a multiple periods

**ARITHMETIC AVERAGE** (Sum of Quarters)

**GEOMETRIC AVERAGE** (Single per Quarter – cumulative +compounding)

	0	1	2	3	4
HPR %		10	25	-20	25

**Arithmetic:**

$$(10+25-20+25) / 4 = 10\%$$

**Geometric:**

$$(1+0.10) * (1+0.25) * (1-0.20) * (1+0.25) = (1+r) ^ 4$$

$$r = [(1+0.10) * (1+0.25) * (1-0.20) * (1+0.25)] ^{1/4} - 1$$

$$(1.1*1.25*.80*1.25)^{1/4} - 1 =$$

$$1.375 ^{1/4} - 1 = 8.29\%$$

**Dollar Weighted Return**

	0 (initial)	1	2	3	4
Net CF (\$)	-1	-0.1	-0.5	0.8	1.0

$$1 = +(-0.1 / (1+IRR) + (-0.5 / (1+ IRR) ^2 + (0.8 / (1+ IRR) ^3 + (1.0 / (1+IRR) ^4)$$

**Excel**

IRR	0	1	2	3	4
4.17%	-1	-0.1	-0.5	0.8	1

=IRR (initial investment, cash flows)

$$= 4.17\%$$

**Conventions for Quoting Rates of Return:** Return on Assets with Regular Cash Flow (Mortgage, bonds – semi-annual coupon)

APR (Annual Percentage Rates) – using simple interest approach

$$\text{APR} = \text{per period rate} * \text{Periods per Year}$$

EAR (Effective Annual Rate)

$$1 + \text{EAR} = (1 + \text{Rate per period})^n = [(1 + (\text{APR}/n))^n]$$

$$\text{APR} = (1 + \text{EAR})^{1/n} - 1 * n$$

For continuous compounding,  $1 + \text{EAR} = e^{\text{APR}}$  or  $\text{APR} = \ln(1 + \text{EAR})$

$$\text{EAR} = (1 + \text{Per period Rate})^{\text{number periods}} - 1$$

*Example:*

Suppose we buy T-Bills maturing in one month for \$9,900 (on maturity you collect the Face Value \$10,000).

$$\text{HPR} = (\text{Cash Income} + \text{Price Change}) / \text{Initial Price}$$

$$\text{HPR} = 100/9,900 = 0.0101 = 1.01\%$$

$$\text{APR} = 1.01\% * 12 \text{ (annualized)} = \underline{\underline{12.12\%}}$$

$$\text{EAR} \rightarrow 1 + \text{EAR} = (1.0101)^{12} = 1.1282$$

$$\text{EAR} = 1.1282 - 1 = .1282 = \underline{\underline{12.82\%}}$$

## RISK AND RETURN PREMIUMS

## HOW DO WE QUANTIFY RISK?????

Scenarios Analysis and Probability Distributions

	Scenarios	Probability	HPR	WAHPR (probability*HPR)
Boom Growth	1	0.25	44%	11.00%
Normal Growth	2	0.5	14%	7.00%
Recession Growth	3	0.25	-16%	-4.00%
				14.00%

**HOW DO WE QUANTIFY THE UNCERTAINTY OF INVESTMENT???**

To summarize risk with a single number we find **VARIANCE**, the expected value of the **squared Deviation for the mean**, first. (I.e. the expected value of the squared “surprise”: across scenarios.)

$$\text{Var. (r)} = \sigma^2 = \sum p(s) [r(s) - E(r)]^2$$

**VARIANCE - DEFINITION**

The Variance (which is the square of the standard deviation, ie:  $\sigma^2$ ) is defined as:

**The average of the squared differences from the Mean.**

In other words, follow these steps:

1. Work out the Mean (the simple average of the numbers)
2. Now, for each number subtract the Mean and then square the result (the squared difference).
3. Then work out the average of those squared differences.

- Squaring each difference makes them all positive numbers (to avoid negatives reducing the Variance)

- And it also makes the bigger differences stand out. For example  $100^2=10,000$  is a lot bigger than  $50^2=2,500$ .

- But squaring them makes the final answer really big, and so un-squaring the Variance (by taking the square root) makes the Standard Deviation a much more useful number.

Variance = Squared Sigma

**STANDARD DEVIATION DEFINITION:** The Standard Deviation ( $\sigma$ ) is a measure of how spreads out numbers are. (Note: Deviation just means how far from the normal). So, using the

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Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small.

	Scenarios	Probability	HPR	WAHPR (probability*HPR)	Variance [probability*(HPR-sum of WAHPR)^2]
Boom Growth	1	0.25	44%	11.00%	225.00
Normal Growth	2	0.5	14%	7.00%	0.00
Recession Growth	3	0.25	-16%	-4.00%	225.00
				<b>14.00%</b>	450.00
				St. Dev =	<b>21.21%</b>

$$SD(r) = \sigma = \sqrt{\text{Var}(r)}$$

$$E(r) = (0.25 * 44\%) + (0.50 * 14\%) + (0.25 * -16\%) = \underline{14\%}$$

$$\text{Sigma}^2 = 0.25 (44 - 14)^2 + 0.50 (14 - 14)^2 + 0.25 (-16 - 14)^2 = 450$$

$$\text{And so the SD sigma} = \sqrt{450} = \underline{21.21\%}$$

## EXAMPLE - table 5.2

Current Price=	23.50						
	Scenarios	Probability	End of yr Price	Dividends	HPR %	WAHPR	Variance
High Growth	1	0.35	\$ 35.00	\$ 4.40	67.66	23.68	591.41
Normal Growth	2	0.30	\$ 27.00	\$ 4.00	31.91	9.57	8.62
No Growth	3	0.35	\$ 15.00	\$ 4.00	(19.15)	(6.70)	731.04

$$E(r) = \underline{26.55} \quad 1,331.07$$

$$\text{HPR} = (\text{End of the year Price} - \text{Current Price} + \text{Div}) / (\text{Current Price})$$

$$\text{StDev} = \underline{36.48}$$

$$\text{Standard Deviation} = \text{Sq Rt of } V$$

$$\text{Variance} = 0.35 * (67.66 - 26.55)^2 + .30 * (31.91 - 26.55)^2 + .35 * (-19.5 - 26.55)^2$$

**RISK PREMIUM Vs RISK AVERSION (Risk Appetite)**

We measure the “Reward” or the difference between the expanded HPR or the Index stock fund and the risk – free rate

HPR – Risk Free Rate = Premium

14% - 6 % = 8%

**VOLATILITY vs RETURN – Relationship**

Sharpe Ratio:

Risk Premium over the Standard Deviation of portfolio excess return

$(E(r_p) - r_f) / \sigma$
---------------------------

8% / 20% = 0.4x. A higher Sharpe ratio indicates a better reward per unit of volatility, in other words, a more efficient portfolio

Sharpe Ratio is more useful for ranking portfolios - it is not valid for individual assets – is useful across Asset Classes.

**HISTORICAL RECORD OF RETURNS – TABLE 5.3 – Panel B**

To calculate average returns and standard deviations from historical data, let’s compute these statistics for the returns on the S&P 500 portfolio using five years of data from the table (5.4)

Example 5.4

Year	ROR	Deviation from Average Return	Deviation From Avg return Squared (^2)
1	16.90%	0.22%	0.05
2	31.30%	14.62%	213.74
3	-3.20%	-19.88%	395.21
4	30.70%	14.02%	196.56
5	7.70%	-8.98%	80.64
	<u>83.40%</u>		<u>886.21</u>

observations (n) = 5

Average ROR = 16.68% = 83.40 / 5

Var = 886.21 / (5-1) = 221.552 = 886.21 / (5-1)

Standard Deviation= 14.88% = SQRT 221.552

INFLATION, NOMINAL AND REAL RATES OF RETURN

Nominal Rate of Return (R) = 10%

Inflation (i) = 6.0%

$r = R - i$
-------------

Real Rate of Return (r) (Approximation) = 4.0%

Real Rate of Return (Exact) = $r = (R-i) / (1+i)$
---

Example

Invest in one-year CD for 8.0%. Inflation is 5.0%. Find the approximate and exact Real Rate of Return:

Approximate  $r = 8.00\% - 5.00\% = 3.00\%$

Exact  $r = (8.00\% - 5.00\%) / (1+5.00\%) = 2.86\%$

EQUILIBRIUM NOMINAL RATE OF INTEREST

Fisher Equation .....  $R = r + E(i)$  .....Nominal rate ought to increase one for one with increase of expected inflation

ASSET ALLOCATION ACROSS RISKY AND RISK FREE PORTFOLIOS

Percentage across

Total Portfolio = **300,000**

Cash	90,000	30%
Stocks	210,000	70%
Total	<u>300,000</u>	

Stocks		of total Portfolio	of total Stocks
S&P 500 Index	113,400	37.800%	54.000%
Fidelity Invest	96,600	32.200%	46.000%
	<u>210,000</u>	<u>70.000%</u>	<u>100.000%</u>
Cash	90,000	30.000%	
	<u>300,000</u>	<u>100.000%</u>	



**Portfolio Expected Return and Risk**

- + Optimal Risky Portfolio (P)
- + Proportion of the Investment budget (Y) to be allocated to it.
- + The remaining portion (1-Y) is to be invested in the Risk-free Asset (rf)
- + Actual risk rate of return by rp on P by E(rp) and Standard Deviation  $\sigma_p$
- + The rate on risk-free asset is denoted as rf
- + The portfolio return is E(rc)

$$\begin{aligned}
 E(rp) &= 15\% \\
 \sigma &= 22\% \\
 R_f &= 7\% \\
 E(rp) - r_f &= 8\%
 \end{aligned}$$

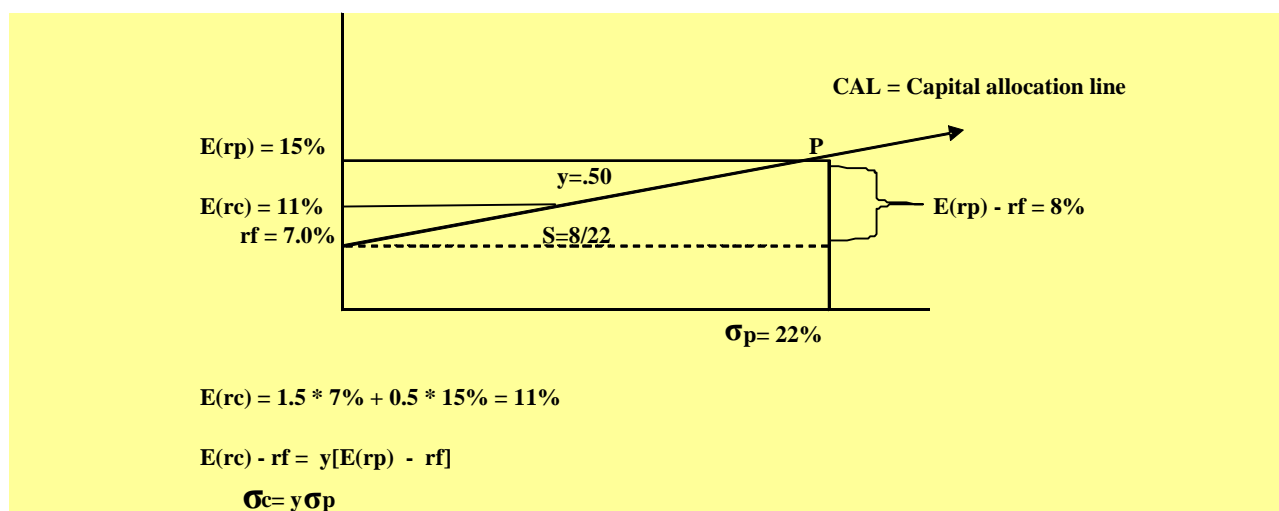
**Let's start with two extreme cases**

1. If y=1 (all of the portfolio in the risk asset)

$$\begin{aligned}
 E(rp) &= 15\% \\
 \sigma_p &= 22\%
 \end{aligned}$$

2. If y=0 (none of the portfolio in the risk asset)

$$\begin{aligned}
 r_f &= 7\% \\
 \sigma_p &= 0\%
 \end{aligned}$$



The Capital Allocation Line (CAL)

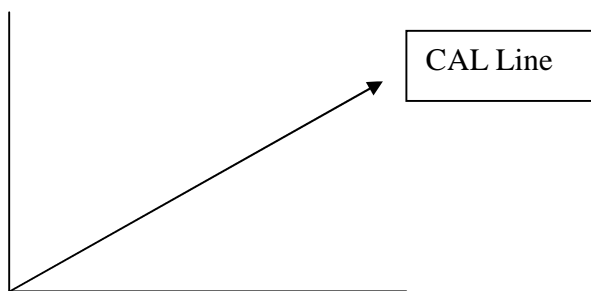
Different values of Y (risky portfolio)

The slope (s) of the CAL equals the increase in expected return that an investor can obtain per unit of additional standard deviation.

THE REWARD-TO-VOLATILITY RATIO (Sharpe Ratio)

	Exp. Return	Risk Premium	Standard Dev.	Sharpe Ratio
Portfolio P.	15%	8%	22%	8/22 = 0.36
Portfolio C.	11%	4%	11%	4/11 = 0.36

Plot on CAL the Sharpe Ratio is the same



$R_f = 7\%$  .... If the investor can borrow at (risk free) rate of  $r_f=7.0\%$ , then he/she can construct a complete portfolio that plot on the CAL line to the right of P where  $y > 1$

Example:

Originally have \$300,000 – borrows additional \$120,000 = \$420,000 invested at y risk

This is a levered position in risky assets

$y = 420,000 / 300,000 = 1.4x$  and  $1-y = - 0.4$ , reflecting a short position in the risk-free assets – a borrowing position = 7.0%

The portfolio rate of return is

$$E(r_c) = 7 + (1.4 * 8) = 18.2\%$$

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Your income = \$63,000 (15% of \$420,000) and pay \$8,400 (7% of 120,000) interest

$$\$63,000 - 8,400 = 54,600$$

$$54,600 / 300,000 = 18.2\%$$

Sharpe Ratio:

$$\sigma_i = 1.4 * 22 = 30.8$$

$$S = (E(r_i) - r_f) / \sigma_i = (18.2 - 7.0) / 30.8 = 11.2 / 30.8 = 0.36$$