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# LECTURE 3

## Bond Portfolio (Chapter 11)

Interest Rate Sensitivity – Calculating Duration and Convexity

$$D_{Mac} = \frac{\sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} t}{V_B}$$

$$C = \frac{\frac{1}{(1+i)^2}\left[\sum_{t=1}^{N} \frac{CF_t}{(1+i)^t}\left(t^2+t\right)\right]}{V_B}$$

Duration: is a measure of the sensitivity of the asset's price to interest rate movements. It broadly corresponds to the length of time before the asset is due to be repaid. This **duration** is equal to the ratio of the percentage reduction in the bond's price to the percentage increase in the redemption yield of the bond (or vice versa) (Lamda)

The standard definition of duration is Macaulay duration, the PV-weighted time to receive each cash flow, defined as:

Weighted Average Wt = [cf / (1 +y)^t ] / Bond Price

Y = yield to maturity
T=time

D = Σ t x Wt

| 100 | K | L | M | N | O | P | Q |
|---|---|---|---|---|---|---|---|
| 101 | **Duration** | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 102 | | | | | | | |
| 103 | Int.Rate = | 10% | | | | | P x 0 |
| 104 | | | Time until | Payment | PV of Pmt | % | Duration |
| 105 | | | Payments | | DR = 10% | Weight | |
| 106 | 8% coupon bond | | **1** | **80** | **72.727** | 7.65% | 0.0765 |
| 107 | | | **2** | 80 | **66.116** | 6.96% | 0.1392 |
| 108 | | | **3** | 1080 | **811.420** | 85.39% | 2.5617 |
| 109 | | | | | 950.263 | 100.00% | 2.7774 |
| 110 | | | | | | | |
| 111 | | Duration | | | | | |
| 112 | | | | | | | |
| 113 | Zero Bond | will be 3 years | | | | | |
| 114 | | | | | | | |
| 115 | | | | | | | |
| 116 | | | | | | | |

Durationis a key concept in bond portfolio management for at least 3 reasons:

1. It's a simple summary measure of the effective average maturity of the portfolio
2. It turns out to be an essential tool in immunizing portfolios from interest rate risk.
3. Duration is the measurement of the interest rate sensitivity of a bond portfolio.

Convexity
**convexity** is a measure of the sensitivity of the duration of a bond to changes in interest rates. There is an inverse relationship between convexity and sensitivity - in general, the higher the convexity, the less sensitive the bond price is to interest rate shifts, the lower the convexity, the more sensitive it is.

Duration is a linear measure or 1st derivative of how the price of a bond changes in response to interest rate changes. As interest rates change, the price is not likely to change linearly, but instead it would change over some curved function of interest rates. The more curved the price function of the bond is, the more inaccurate duration is as a measure of the interest rate sensitivity.

Convexity is a measure of the curvature or 2nd derivative of how the price of a bond varies with interest rate, i.e. how the duration of a bond changes as the interest rate changes.

$$\Delta P / P = - D \times \Delta y$$

| 100 | K | L | M | N | O | P | Q | R | S | |
|---|---|---|---|---|---|---|---|---|---|---|
| 101 | **MACAULAY DURATION AND CONVEXITY** | | | | | | | | | |
| 102 | **Sensitivity to interest rate movements** | | | | | | | **IRR=** | **10.0000%** | |
| 103 | | | | *=-PV(M108/M109,M107*M109,M106*M105/M109,M105)* | | | | | | ($820.74) |
| 104 | Bond Price | | $875.38 | | | **If Yield Changes By** | | **1.00%** | | |
| 105 | Face Value | | **1,000** | | | Bond Price Will Change By | | -54.63 | -6.24% | =+R105/M104 |
| 106 | Coupon Rate | | **8.00%** | | | | | | =-(M104+PV((M108+R104)/M109,M107*M109,M106*M105/M109,M105)) | |
| 107 | Life in Years | | **10** | | | Modified Duration Predicts | | -57.03 | =(-M112*R104*M104) | |
| 108 | Yield | | **10.00%** | | | Convexity Adjustment | | 2.47 | *=0.5*M113*R104^2*M104* | |
| 109 | Frequency | | **2** | | | Total Predicted Change | | -54.56 | *=+R107+R108* | |
| 110 | | | | | | | | | | |
| 111 | Macaulay Duration | | 6.84 | *=+P137/M104/M109* | | Actual New Price | | $820.74 | *=-PV((M108+Q104)/M109,M107*M109,M106*M105/M109,M105)* | |
| 112 | Modified Duration | | 6.51 | *=+M111/(1+M108/M109)* | | Predicted New Price | | $820.82 | *=+M104+R109* | |
| 113 | Convexity | | 56.49 | *=+S137/M104/M109^M109* | | Difference | | $0.08 | *=+R112-R111* | |
| 114 | | | | | | | | | | |
| 115 | **Period** | **Cash Flow** | **PV Cash Flow** | **Weighted** | **Duration Calc Method 1** | **Duration Calc Method 2** | **PV of pv(CF)** | **Factor years** | **Convexity Calc** | |
| 116 | 0 | ($875.38) | | | | | | | | |
| 117 | 1 | 40.00 | 38.10 | 4.352% | 0.04352 | 38.10 | 34.554 | 2.000 | 69.11 | |
| 118 | 2 | 40.00 | 36.28 | 4.145% | 0.08289 | 72.56 | 32.908 | 6.000 | 197.45 | |
| 119 | 3 | 40.00 | 34.55 | 3.947% | 0.11842 | 103.66 | 31.341 | 12.000 | 376.09 | |
| 120 | 4 | 40.00 | 32.91 | 3.759% | 0.15037 | 131.63 | 29.849 | 20.000 | 596.97 | |
| 121 | 5 | 40.00 | 31.34 | 3.580% | 0.17901 | 156.71 | 28.427 | 30.000 | 852.82 | |
| 122 | 6 | 40.00 | 29.85 | 3.410% | 0.20459 | 179.09 | 27.074 | 42.000 | 1,137.09 | |
| 123 | 7 | 40.00 | 28.43 | 3.247% | 0.22732 | 198.99 | 25.784 | 56.000 | 1,443.92 | |
| 124 | 8 | 40.00 | 27.07 | 3.093% | 0.24742 | 216.59 | 24.557 | 72.000 | 1,768.07 | |
| 125 | 9 | 40.00 | 25.78 | 2.946% | 0.26510 | 232.06 | 23.387 | 90.000 | 2,104.85 | |
| 126 | 10 | 40.00 | 24.56 | 2.805% | 0.28052 | 245.57 | 22.273 | 110.000 | 2,450.08 | |
| 127 | 11 | 40.00 | 23.39 | 2.672% | 0.29388 | 257.26 | 21.213 | 132.000 | 2,800.10 | |
| 128 | 12 | 40.00 | 22.27 | 2.544% | 0.30533 | 267.28 | 20.203 | 156.000 | 3,151.62 | |
| 129 | 13 | 40.00 | 21.21 | 2.423% | 0.31503 | 275.77 | 19.241 | 182.000 | 3,501.80 | |
| 130 | 14 | 40.00 | 20.20 | 2.308% | 0.32310 | 282.84 | 18.324 | 210.000 | 3,848.14 | |
| 131 | 15 | 40.00 | 19.24 | 2.198% | 0.32970 | 288.61 | 17.452 | 240.000 | 4,188.45 | |
| 132 | 16 | 40.00 | 18.32 | 2.093% | 0.33493 | 293.19 | 16.621 | 272.000 | 4,520.86 | |
| 133 | 17 | 40.00 | 17.45 | 1.994% | 0.33892 | 296.68 | 15.829 | 306.000 | 4,843.78 | |
| 134 | 18 | 40.00 | 16.62 | 1.899% | 0.34177 | 299.17 | 15.076 | 342.000 | 5,155.85 | |
| 135 | 19 | 40.00 | 15.83 | 1.808% | 0.34357 | 300.76 | 14.358 | 380.000 | 5,455.92 | |
| 136 | 20 | 1,040.00 | 391.97 | 44.777% | 8.95533 | 7,839.30 | 355.524 | 420.000 | 149,320.02 | |
| 137 | | **Total** | 875.38 | 100% | 13.68074 | 11,975.81 | 793.99356 | | 197,783.01 | |
| | | | | | 6.84037 | | | | | |

**Bond Terminology**

**Accrued Interest**

Accrued interest is the interest that has been earned, but not yet been paid by the bond issuer, since the last coupon payment. Note that interest accrues equally on every day during the period. That is, it does not compound. So, halfway through the period, you will have accrued exactly one-half of the period's interest payment. It works the same way for any other fraction of a payment period.

**Banker's Year**

A banker's year is 12 months, each of which contains 30 days. Therefore, there are 360 (not 365) days in a banker's year. This is a convention that goes back to the days when "calculator" and "computer" were job descriptions instead of electronic devices. Using 360 days for a year made calculations easier to do. This convention is still used today in some calculations such as the Bank Discount Rate that is used for discount (money market) securities.

**Bond**

A bond is a debt instrument, usually tradable, that represents a debt owed by the issuer to the owner of the bond. Most commonly, bonds are promises to pay a fixed rate of interest for a number of years, and then to repay the principal on the maturity date. In the U.S. bonds typically pay interest every six months (semi-annually), though other payment frequencies are possible. Bonds are issued by corporations, banks, state and local governments (municipal bonds), and the federal government (Treasury Notes and Bonds).

**Call Date**

Some bonds have a provision in the indenture that allows for early, forced, redemption of the bond, often at a premium to its face value. Bonds that have such a feature usually have a series of such dates (typically once per year) at which they can be called. This series of dates is referred to as the call schedule.

**Call Premium**

The extra amount that is paid by a bond issuer if the bond is called before the maturity date. This is a sweetener that is used to make callable bonds attractive to investors, who would otherwise prefer to own non-callable bonds.

**Clean Price**

The "clean price" is the price of the bond excluding the accrued interest. This is also known as the quoted price.

**Coupon Payment**

The is the actual dollar amount that is paid by the issuer to the bondholders at each coupon date. It is calculated by multiplying the coupon rate by the face value of the bond and then dividing by the number of payments per year.

**Coupon Payment Date**

The specified dates (typically two per year) on which interest payments are made.

**Coupon Rate**

The stated rate of interest on the bond. This is the annual interest rate that will be paid by the issuer to the owners of the bonds. This rate is typically fixed for the life of the bond, though variable rate bonds do exist. The term is derived from the fact that, in times past, bond certificates had coupons attached. The coupons were redeemed for cash payments.

**Current Yield**

A measure of the income provided by the bond. The current yield is simply the annual interest payment divided by the current market price of the bond. The current yield ignores the potential for capital gains or losses and is therefore not a complete measure of the bond's rate of return.

**Day-count Basis**

A method of counting the number of days between two dates. There are several methods, each of which makes different assumptions about how to count. **30/360** (a banker's year) assumes that each month has 30 days and that there are 360 days in a year. **Actual/360** counts the actual number of days, but assumes that there are 360 days in a year. **Actual/Actual** counts the actual number of days in each month, and the actual number of days in a year. In Excel bond functions, 0 signifies 30/360, 1 specifies actual/actual, 2 is actual/360, 3 is actual/365 (which ignores leap days), and 4 represents the European 30/360 methodology.

**Dirty Price**

The "dirty price" is the total price of the bond, including accrued interest. This is the amount that you would actually pay (or receive) if you purchase (or sell) the bond.

**Face Value**

The principal of a bond is the notional amount of the loan. It is also called the **principal** or **par** value of the bond, and represents the amount that will be repaid when the bond matures.

**Indenture**

The legal contract between a bond issuer and the bondholders. The indenture covers such things as the original term to maturity, the interest rate, interest payment dates, protective covenants, collateral pledged (if any), and so on.

**Maturity Date**

The date on which the bond ceases to earn interest. On this date, the last interest payment will be made, and the face value of the bond will be repaid. This is also sometimes known as the **redemption date**.

**Redemption Value**

This is typically the same as the face value of a bond. However, for a callable bond, it is the face value plus the call premium. In other words, this is the entire amount that will be received when the bond is redeemed by the issuer.

**Settlement Date**

The date on which ownership of a security actually changes hands. Typically, this is several days after the trade date. In the US markets, the settlement date is usually 3 trading days after the trade date (this is known as T+3). For bonds, a purchaser begins to accrue interest on the settlement date.

**Term to Maturity**

The amount of time until the bond stops paying interest and the principal is repaid.

**Yield to Call**

Same as yield to maturity, except that we assume that the bond will be called at the next call date. Also known as yield to first call. Frequently, the yield to all call dates is calculated, and then we can find the worst-case, which is known as the yield to worst.

**Yield to Maturity**

The yield to maturity (YTM) of a bond is the compound average annual expected rate of return if the bond is purchased at its current market price and held to maturity. Implicit in the calculation of the YTM is the assumption that the interest payments are reinvested for the life of the bond at the same yield. The YTM is the internal rate of return (IRR) of the bond.

**Yield to Worst**

The lowest of all possible yields for the bond. It is calculated by determining the minimum of the yield to maturity or any of the various yields to call date