BARUCH COLLEGE – DEPARTMENT OF ECONOMICS & FINANCE
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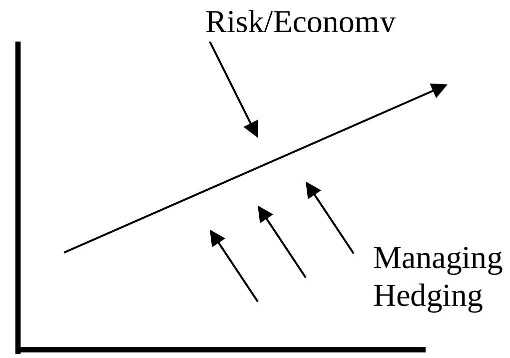
LECTURE 1

FIN 3710 REVIEW



DEFINITIONS:

- Value Creation (Cost < Result) – Investment
- Return Vs Risk - Analysis

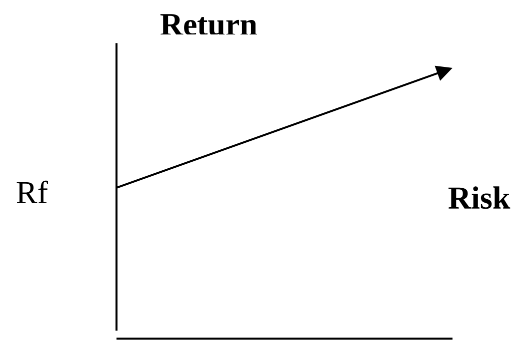
Real Assets Vs Financial Assets (Land/Building Vs Stock/Bonds)

3 Investments Cat.: **Debt (Fixed Income), Equity, Derivatives**

DEBT	EQUITY	DERIVITIVES
Promise to Pay	Ownership	Options/futures
Set Maturities	Not a promise to pay	Bets on movements
Long Term/Short	Downside/Upside	Transfer / Hedge Risk /
• Government/Municipal	Bottom of Waterfall	Insurance on Movements
• Corporate		Swaps/FX/Equity

Investment Factors:
- Return (Expected Return)
- Risk – Quantifying Risk – Volatility/credit/interest/duration/systemic
- Time
- Allocation

Analysis:
- Fundamental
- Technical
- Behavioral



Chapter 5

RISK RETURN –

- Traditionally, when you define return you refer to a bank savings account (risk free) plus a risky portfolio of US stocks. – Today – investors have access to a variety of asset classes, financial engineered investments
- The Book "The Quants" by Scott Peterson – financial engineering – achieving the ALPHA.

HPR = (Ending Price – Beg. Price + Div) / Beg. Price

Example:

Current Price = $100, expected price to increase to $110 in a year. Within the year you are expected to receive $4 dividend, therefore the HPR=(110-100+4)/$100 = 14%

Starwoood Hotels
5-yr stock prices

Excel

IRR	0	1	2	3	4
4.17%	-1	-0.1	-0.5	0.8	1

=IRR (initial investment, cash flows)

= 4.17%

EAR , 1 + EAR = (1.0101) ^12 = 1.1282

EAR = 1.1282 – 1 = .1282 = **12.82%**

RISK AND RETURN PREMIUMS

HOW DO WE QUANTIFY RISK?????

Scenario and Historical Analysis

Scenarios Analysis and Probability Distributions

	Scenarios	Probability	HPR	WAHPR
Boom Growth	1	0.25	44%	11.00%
Normal Growth	2	0.5	14%	7.00%
Recession Growth	3	0.25	-16%	-4.00%
				14.00%

HOW DO WE QUANTIFY THE UNCERTAINTY OF INVESTMENT???

To summarize risk with single number we find before the **VARIANCE** as the expected value of the **squared Deviation for the mean.** i.e. the expected value of the squared “surprise: across scenarios.

$$\text{Var. (r)} = \sum \hat{}\ 2 = \sum p(s)\ [\ r(s) - E(r)\ )\ \hat{}2$$

VARIANCE - DEFINITION

The Variance (which is the square of the standard deviation, ie: σ^2) is defined as:

The average of the squared differences from the Mean.

In other words, follow these steps:

1. Work out the Mean (the simple average of the numbers)
2. Now, for each number subtract the Mean and then square the result (the squared difference).
3. Then work out the average of those squared differences.

- Squaring each difference makes them all positive numbers (to avoid negatives reducing the Variance)

- And it also makes the bigger differences stand out. For example 100^2=10,000 is a lot bigger than 50^2=2,500.

- But squaring them makes the final answer really big, and so un-squaring the Variance (by taking the square root) makes the Standard Deviation a much more useful number.

Variance = Squared Sigma

STANDARD DEVIATION **_ DEFINITION: The Standard Deviation (σ) is a measure of how spreads out numbers are**. (Note: Deviation just means how far from the normal). So, using the Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small.

	Scenarios	Probability	HPR	WAHPR	Variance
Boom Growth	1	0.25	44%	11.00%	225.00
Normal Growth	2	0.5	14%	7.00%	0.00
Recession Growth	3	0.25	-16%	-4.00%	225.00
				14.00%	450.00
				St. Dev =	**21.21%**

SD (r) = σ = √ Var (r)

E (r) = (0.25 * 44%) + (0.50 * 14%) + (0.25*(-16%)) = **14%**

Sigma ^2 = 0.25 (44 – 14) ^2 + 0.50 (14 – 14) ^2 + 0.25 (-16-14)^2 = 450

And so the SD sigma = √450 = **21.21%**

EXAMPLE - table 5.2

Current Price=	**23.50**						
	Scenarios	**Probability**	**End-of the yr Price**	**Dividends**	**HPR %**	**WAHPR**	**Variance**
High Growth	1	0.35	$ 35.00	$ 4.40	67.66	23.68	591.41
Normal Growth	2	0.30	$ 27.00	$ 4.00	31.91	9.57	8.62
No Growth	3	0.35	$ 15.00	$ 4.00	(19.15)	(6.70)	731.04

E (r) = **26.55** 1,331.07

HPR = (End of the year Price - Current Price + Div) / (Current Price)

StDev = **36.48**

Standard Deviation = Sq Rt of V

Variance = 0.35 * (67.66 - 26.55) ^ 2 + .30 *(31.91 - 26.55) ^ 2 + .35 * (-19.5 - 26.55) ^ 2

RISK PREMIUM Vs RISK AVERSION (Risk Appetite)

We measure the "Reward" or the difference between the expanded HPR or the Index stock fund and the risk – free rate

HPR – Risk Free Rate = Premium

14% - 6 % = 8%

VOLATILITY Vs RETURN – Relationship

Sharpe Ratio:

Risk Premium over the Standard Deviation of portfolio excess return

$$(E(r\ p) - r\ f\ ) / \sigma$$

8% / 20% = 0.4x. A higher Sharpe ratio indicates a better reward per unit of volatility, in other words, a more efficient portfolio

Sharpe Ratio is more useful for ranking portfolios - it is not valid for individual assets – is useful across Asset Classes.

HISTORICAL RECORD OF RETURNS – **TABLE 5.3 – Panel B**

To calculate average returns and standard deviations from historical data, let's compute these statistics for the returns on the S&P 500 portfolio using five years of data from the table (5.4)

Example 5.4

Year	ROR	Deviation from Average Return	Squared (^2)
1	16.90%	0.22%	0.05
2	31.30%	14.62%	213.74
3	-3.20%	-19.88%	395.21
4	30.70%	14.02%	196.56
5	7.70%	-8.98%	80.64
	83.40%		886.21

observations (n) =	5	
Average ROR =	16.68%	= 83.40 / 5
Var = 886.21 / (5-1) =	221.552	= 886.21 / (5-1)
Standard Deviation=	14.88%	= SQRT 221.552

INFLATION, NOMINAL AND REAL RATES OF RETURN

Nominal Rate of Return (R) = 10%
Inflation (i) = 6.0%

$$r = R - i$$

Real Rate of Return (r) (Approximation) = 4.0%

Real Rate of Return (Exact) = $r = (R-i) / (1+i)$

Example
Invest in one-year CD for 8.0%. Inflation is 5.0%. Find the approximate and exact Real Rate of Return:

Approximate R = 8.00% - 5.00% = 3.00%
Exact R = (8.00% - 5.00%)/(1+5.00%) = 2.86%

EQUILIBRIUM NOMINAL RATE OF INTEREST

Fisher Equation ……… R = r + E (i) ....Nominal rate ought to increase one for one with increase of expected inflation

ASSET ALLOCATION ACROSS RISKY AND RISK FREE PORTFOLIOS

Percentage across

Total Portfolio =	**300,000**		
Cash	90,000	30%	
Stocks	210,000	70%	
Total	300,000		
Stocks		of total Portfolio	of total Stocks
S&P 500 Index	113,400	37.800%	54.000%
Fidelity Invest	96,600	32.200%	46.000%
	210,000	70.000%	100.000%
Cash	90,000	30.000%	
	300,000	100.000%	

Portfolio Expected Return and Risk

+ Optimal Risky Portfolio (P)
+ Proportion of the Investment budget (Y) to be allocatedn to it.
+ The remaining portion (1-Y) is to be invested is the Risk-free Asset (F)
+ Actuak risk rate of return by rp on P by E (rp) and Standard Deviation σp
+ The rate on risk-free asset is denoted a rf

E(rp) =	15%
σ =	22%
rf=	7%
E(rp)-rf =	8%

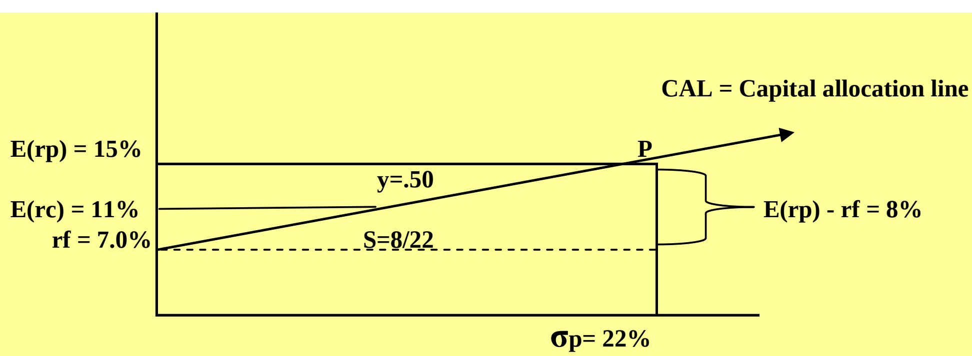
Let's start with two extreme cases

1. if y=1 (all of the portfolio in the risk asset)

E(rp) =	15%
σp=	22%

2. if y=0 (none of the portfolio in the risk asset)

rf=	7%
σp=	0%



E(rc) = 1.5 * 7% + 0.5 * 15% = 11%

E(rc) - rf = y[E(rp) - rf]

$\sigma c = y \sigma p$

The Capital Allocation Line (CAL)

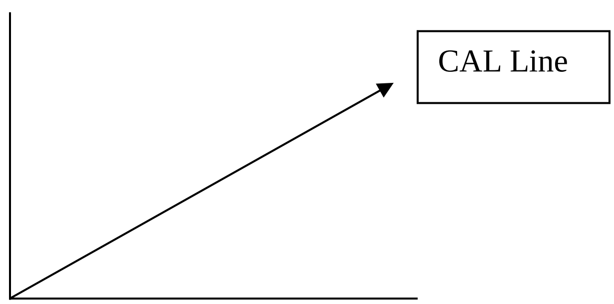
Different values of Y (risky portfolio)

The slope (s) of the CAL equals the increase in expected return that an investor can obtain per unit of additional standard deviation.

THE REWARD-TO-VOLATILITY RATIO (Sharpe Ratio)

	Exp. Return	Risk Premium	Standard Dev.	Sharpe Ratio
Portfolio P.	15%	8%	22%	8/22 = 0.36
Portfolio C.	11%	4%	11%	4/11 = 0.36

Plot on CAL the Sharpe Ratio is the same



Rf = 7% …. If the investor can borrow at (risk free) rate of rf=7.0%, then he/she can construct a complete portfolio that plot on the CAL line to the right of P where y>1

Example:

$300,000 – borrows additional $120,000 = $420,000 invested at y risk

this is a levered position in the risky assets

y = 420,000 / 300,000 = 1.4x and 1-y= - 0.4, reflecting a short position in the risk-free assets – a borrowing position = 7.0%

The portfolio rate of return is

E(rc) = 7 + (1.4 * 8) = 18.2%

Your income = $63,000 (15% of $420,000) and pay $8,400 (7% of 120,000) interest

$63,000 – 8,400 = 54,600

54,600 / 300,000 = 18.2%

Sharpe Ratio:

$\sigma i = 1.4 * 22 = 30.8$

$S = (E (ri) – rt) / \sigma i = (18.2 – 7.0) / 30.8 = 11.2 / 30.8 = 0.36$

Chapter 6:

EFFICIENT DIVERSIFICATION
How investors can construct the best possible risky portfolio – efficient Diversification

"Diversification reduces the variability of portfolio returns"

DIVERSIFICATION AND PORTFOLIO RISK

From one stock to two stocks to three stocks….. sensitivity to external factors (i.e. oil, non-oils stocks) – But even extensive diversification cannot eliminate risk – MARKET RISK

- Other Names for Market risk: Systematic risk, non-diversifiable risk
- The Risk that can be eliminated by diversification is called:
    - Unique Risk
    - Firm-specific risk
    - Non-systematic risk
    - Diversifiable risk

ASSET ALLOCATION

Asset allocation between 2 risky assets

COVARIANCE AND CORRELATION

Relationship between the return of two assets
1. Tandem
2. Opposition

} Depends on the Correlation between the two returns

Use the Economic Scenarios between two asset classes (Stocks and Bonds)

PERFORMANCE SCENARIOS

Scenario (S)	Probability (p)	Stocks (s): ROR % (r_s)	Stocks (s): p * r_s %	Stocks (s): Deviation for Exp. Ret. (Dev.)	Stocks (s): Square Deviation (SD) Dev^2	Stocks (s): p * SD	Bonds (b): ROR % (r_b)	Bonds (b): p * r_b %	Bonds (b): Deviation for Exp. Ret. (Dev.)	Bonds (b): Square Deviation (SD) Dev^2	Bonds (b): p * SD
Recession (Sr)	30.0%	-11.00	-3.30	-21.00	441.00	132.30	16.00	4.80	10.00	100.00	30.00
Normal (Sn)	40.0%	13.00	5.20	3.00	9.00	3.60	6.00	2.40	0.00	0.00	0.00
Boom (Sb)	30.0%	27.00	8.10	17.00	289.00	86.70	-4.00	-1.20	-10.00	100.00	30.00
	100.0%		10.00 %		Variance=	222.60		6.00 %		Variance=	60.00
					SD =	14.92 %				SD =	7.75 %

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation

Stocks (As) =	**60%**
Bonds (Ab) =	**40%**

($As * r_s$) + ($Ab * r_b$)

Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	30.0%	-0.2	-0.06	-8.60	73.96	22.19
Normal (Sn)	40.0%	10.2	4.08	1.80	3.24	1.30
Boom (Sb)	30.0%	14.6	4.38	6.20	38.44	11.53
	100.0%		8.40 %		Variance=	35.02
					SD =	5.92 %

COVARIANCE & CORRELATION

Scenario (S)	Probability (p)	Stocks (Deviation from the mean)	Bonds (Deviation from the mean)	Ds * Db	Covariance [p * (Ds*Db)
Recession (Sr)	30.0%	-21.00	10.00	-210.00	-63.00
Normal (Sn)	40.0%	3.00	0.00	0.00	0.00
Boom (Sb)	30.0%	17.00	-10.00	-170.00	-51.00
	100.0%			Covariance=	-114.00
				Correlation Coefficient =	-0.99

The Covariance is calculated in a manner similar to the Variance. Instead of measuring the typical difference of an asset return from its expected value.

Instead measure the extent to which the variation in the returns of the two assets tend to reinforce or offset each other

COVERIANCE

$$Cov (rs.rb) = \Sigma\ p (i) [rs (i) – avg\ rs] [rb (i) – Avg\ rb]$$

Rs = return on the stock
Rb = return on the bond
P (i) = expected portfolio return

CORRELATION COEFFICIENT

$$P_{sb} = Cov (rs,rb) / \sigma s . \sigma b$$

Psb = portfolio of Stocks and bonds
σs = Standard Deviation of s
σb = Standard Deviation of b

THE 3 RULES OF TWO-RISKY ASSET PORTFOLIOS

Rule 1: ROR of the portfolio is weighted average of the returns

$$rp = Wb. rb + Ws. rs$$

Rule 2: Expected ROR or the portfolio

$$E(rp) = Wb . E(rb) + Ws . E(rs)$$

Rule 3: Variance of ROR or two-risky asset portfolio.

$$\sigma p^2 = (Wb.\sigma b)^2 + (Ws.\sigma s)^2 + \mathbf{2 (Wb.\sigma b) (Ws.\sigma s). Pbs}$$

Pbs is the correlation between the return on stock and bonds

Example: 100% Bonds, then decide to shift to 50% of bonds and 50% of stock

Input Data:

E(rb) = 6.0%
E(rs) = 10%
σb= 12%
σs= 25%
Pbs = 0
Wb=0.5
Ws=0.5

$\sigma p^2=(0.5*12)^2 + (0.5*25)^2 + 2(0.5*12)(0.5*25)*0$

σp = SqRt of 192.25 = 13.87%

If we averaged the 2 standard deviations of each asset class we will have incorrectly predicted an increase in the portfolio's SD (25 + 12)/2 = 18.5% showing an increase of 6.5% when moving from all bond portfolio to half/half bond/stock. The actuality is that the SD movement is much lower to 13.87% (as is calculated above) or 1.87% from all bond portfolio SD of 12.0% - **SO THE GAIN OF DIVERSIFICATION CAN BE SEEN AS FULL 6.50 – 1.87 = 4.62%.**

*If weights 0.75 and 0.25 then (0.75*6) + (0.25*10) =* ***7.0% expected returns***

*Variance = (0.75*12) ^2 + (0.25*25)^2 + 2(0.75*12) (0.25*25) *0*

SqRt of 120 **= *10.96%***

Check page 159 – Graph and Table at rs=10, rb=6, σs=25, σb=12 at different weights

Parameters

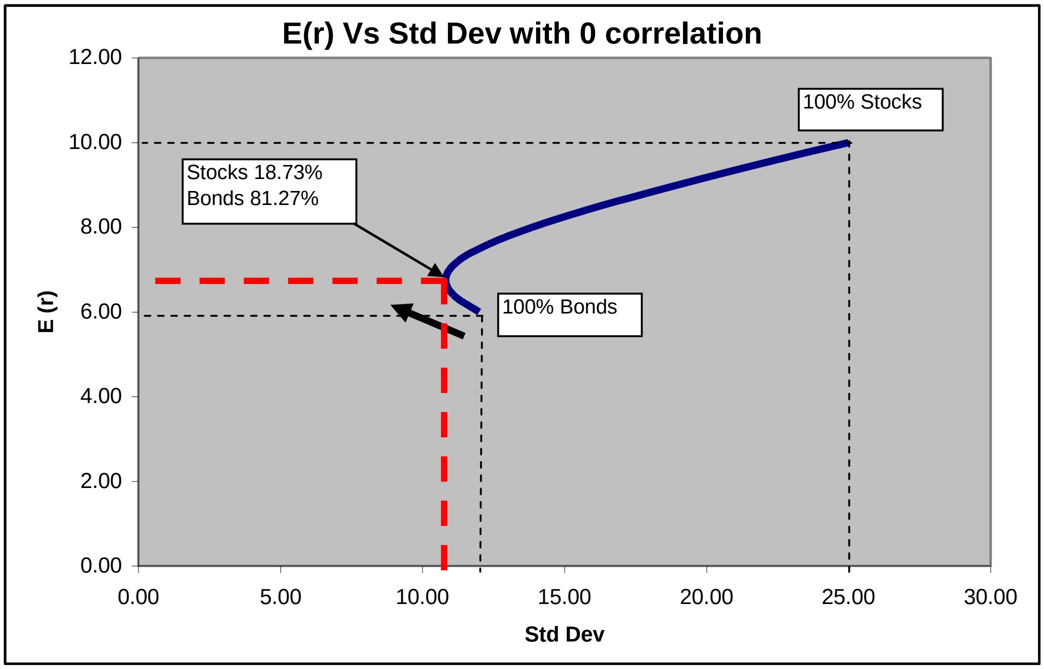
E (rs) =	**10**
E (rb) =	**6**
σs =	**25**
σb=	**12**
Psb =	**0**

Portfolio Weights		**Exp Return**	**Std Dev.**
Ws	**Wb**	**E(rp) %**	**σp %**
0.0	1.0	6.00	12.00
0.1	0.9	6.40	11.09
0.2	0.8	6.80	10.82
0.3	0.7	7.20	11.26
0.4	0.6	7.60	12.32
0.5	0.5	8.00	13.87
0.6	0.4	8.40	15.75
0.7	0.3	8.80	17.87
0.8	0.2	9.20	20.14
0.9	0.1	9.60	22.53
1.0	0.0	10.00	25.00

Minimum Variance

Stocks	18.7256%
Bonds	81.2744%

Ws=(σb^2 - σb σs p) / (σs^2 + σb^2 - 2*σb σs p)
Wb=1-Ws



<u>The Mean – Variance Criterion</u>

Investors Desire portfolios to lie to the Nortwest (Graph) – with higher return and lower Standard Deviation (Risk)

Let's assume Portfolio A is said to dominate portfolio B if all investors prefer A over B. This will be the case that has the highest Return and lost Variance

$E(rA) \geq E(rB)$ and $\sigma A \leq \sigma B$

If we graph the relationship PA will be to the Northwest of PB

WHAT ARE THE IMPLICATIONS OF PERFECT POSITIVE CORRELATION BETWEEN BONDS & STOCKS??

Let's say the correlation is 1 or Pbs = 1 (so far we used 0 correlation)

Pbs = 1

σp^2 = Wb^2 σb ^2 + Ws^2 σs^2 + 2 Wb σb Ws σs * 1 = Wb.σb + Ws.σs)

so if Pb = 1 then σp = Wb.σb + Ws.σs

we learned if

Pb = 0 then σp = SqRt of (Wb.σb)^2+ (Ws.σs)^2

Example we were using (σs = 25, σb = 12)

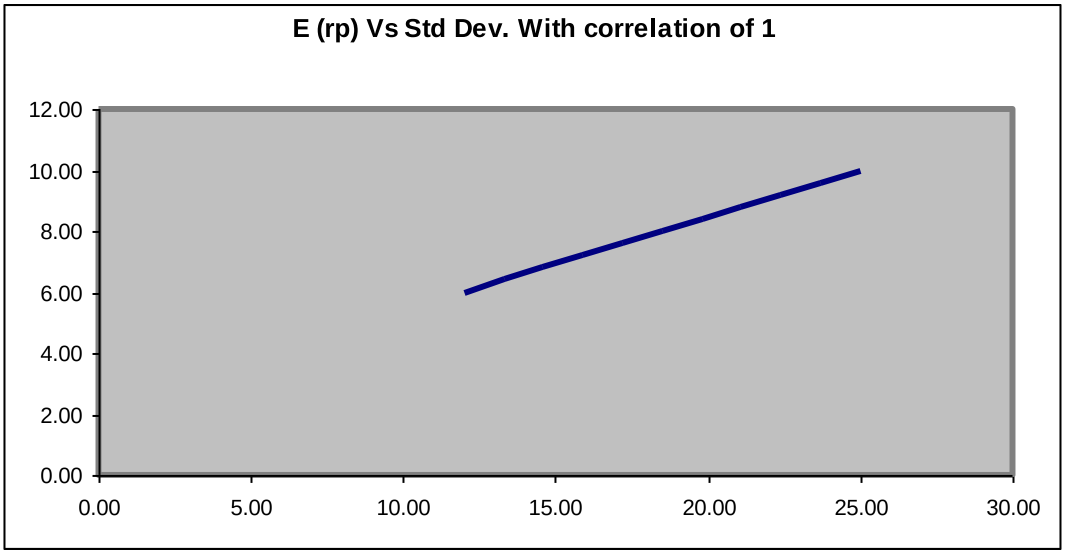
σp= (.50 * 12) + (.50 * 25) = 18.75% …. If Pbs = 1, straight average – No gain for diversification, where Pbs = 0 we calculated previously that the σp = 13.87% .

Graph of Pbs = 1 and Pbs = 0 and in between

With Correlation = 1

Psb = 1

Portfolio Weights		Std Dev.	Exp Return
Ws	**Wb**	**σp %**	**E(rp) %**
0.0	1.0	12.00	6.00
0.1	0.9	13.30	6.40
0.2	0.8	14.60	6.80
0.3	0.7	15.90	7.20
0.4	0.6	17.20	7.60
0.5	0.5	18.50	8.00
0.6	0.4	19.80	8.40
0.7	0.3	21.10	8.80
0.8	0.2	22.40	9.20
0.9	0.1	23.70	9.60
1.0	0.0	25.00	10.00



Use Extreme Example where **Pbs = -1**

σp^2 = (Wb.σb – Ws.σs)^2

or σp = ABS Wb.σb – Ws.σs

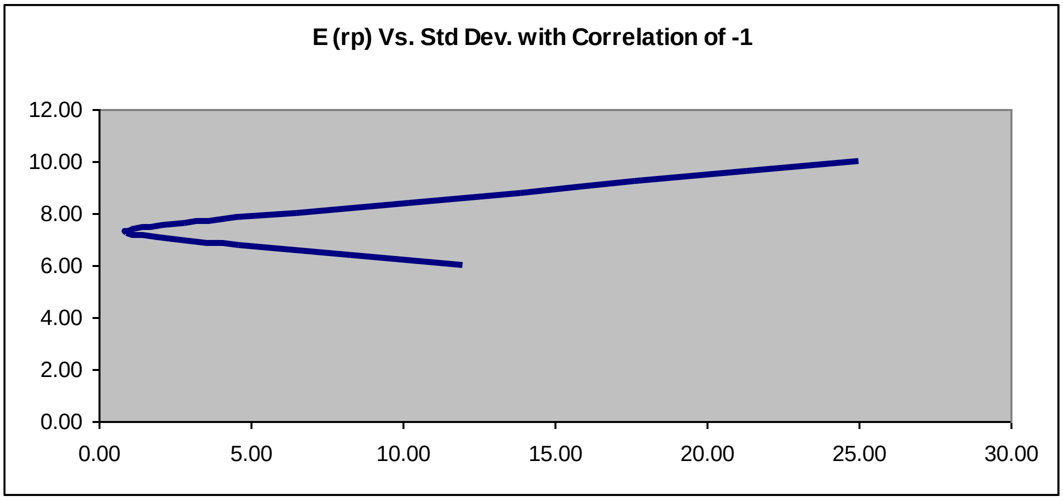
(using ABS or absolute because there is no negative standard deviation)

using our example = .50*12 - .50*25 = Abs 6.5%

With Correlation = -1

Psb = -1

Portfolio Weights		Std Dev.	Exp Return
Ws	**Wb**	**σp %**	**E(rp) %**
0.0	1.0	12.00	6.00
0.1	0.9	8.30	6.40
0.2	0.8	4.60	6.80
0.3	0.7	0.90	7.20
0.4	0.6	2.80	7.60
0.5	0.5	6.50	8.00
0.6	0.4	10.20	8.40
0.7	0.3	13.90	8.80
0.8	0.2	17.60	9.20
0.9	0.1	21.30	9.60
1.0	0.0	25.00	10.00



THE OPTIMAL RISKY PORTFOLIO W A RISK-FREE ASSET

Let's add Risk Free in our portfolio (bringing what we discussed before regarding CAL line)

Historical Correlation between Bonds and Stocks is 0.20

T-Bills = 5.0% (risk free)

GRAPH introducing the CAL in our previous Graph of Bonds and Stock

Using the minimum (point A) on a .20 correlation between bonds and stock. We were given the minimum weights at Wb= 87.06% and Ws = 12.94% so PA expects to return 6.52% and σA is 11.54%calculated as follows:

rA = (.8706 * 6) + (.1294 * 10) = 6.52

σA=(.8706 * 12) ^2 + (.1294 * 25) ^2 = 11.54%

Sharpe Ratio is SA = (E (rA) – rf) / σA = (6.52 – 5) / 11.54 = 0.13

Now consider the CAL uses portfolio B instead of A. Portfolio B consists of 80% Bonds and 20% Stock, then rbs = 6.80%, σbs = 11.68% then,

SB = (6.80 – 5) / 11.68 = 0.15

SB – SA = 0.02

This implies that portfolio B provides 2 extra basis points (0.02%) of expected return for every percentage point (1.0%) increased in Standard Deviation (Risk)
The higher Sharpe Ratio of B means that its capital allocation line (CAL) it's steeper than A, therefore, CAL(B) plots above CAL(A).
In other words, combination of portfolio B and the risk-free asset provide a higher expected return for any level of risk (SD) than combination of portfolio A and the risk free risk.

GOAL = **CAL NEED TO REACH TANGENCY (GRAPH) FOR OPTICAL RISKY PORTFOLIO**

Graph 6.6, page 166

Solution for maximizing of the Sharpe Ratio:

Wb = [(E(rb) – rf).σs^2 – (E9rs) – rf).σb.σs.Pbs] / [(E (rb) – rf) σs^2 + (E (rs) – rf).σb^2 – rf + E (rs) – rf.σb.σs.Pbs

Ws = 1- Wb

BUILDING A PORTFOLIO WITH RISK FREE, STOCK, AND BONDS

Assume we want to invest 45% of our portfolio in Risk Free assets = 55% is in a risky portfolio between bonds (50%) and stocks (50%),

We find the CAL with our optimal portfolio (o) in a slope – Lets say:

Pro = 8.68% and σ0=17.97%, Wb = 32.99% and Ws = 67.01% from the long formula above.

So = 8.68 – 5 / 17.97 = 0.20

E(rc) = 5 + 0.55 *(8.68 – 5) = 7.02%
σc = 0.55 * 17.97 = 9.88%

Wrf = 45%
Wb = 0.3299 * .55 = 18.14%
Ws = 0.6701 * .55 = 36.86%

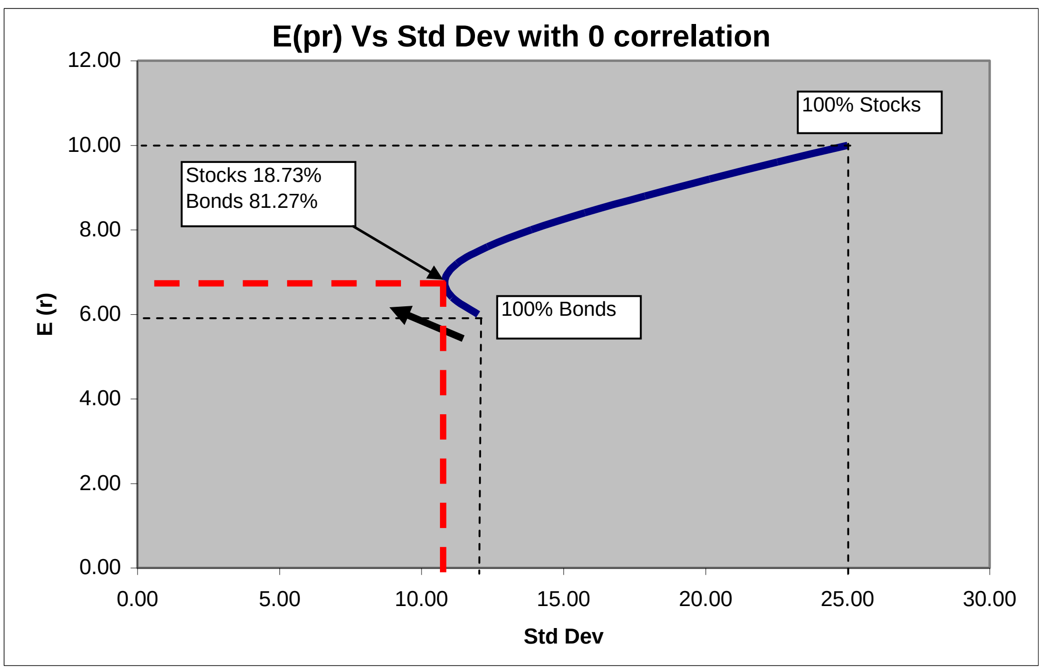
REVIEW – CHAPTER 6

THE EFFICIENT FRONTER OF RISKY ASSETS

3 STEPS:

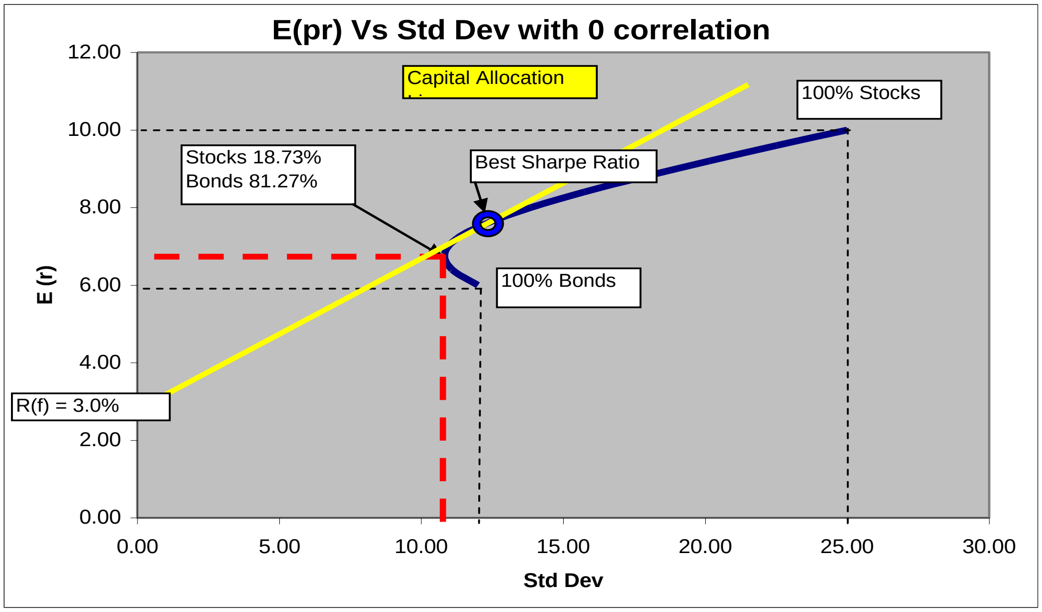
STEP 1:
Identify the best possible or most efficient risk-return combination available from the universe of risky assets (Plot them on Return/Standard Deviation Graph)

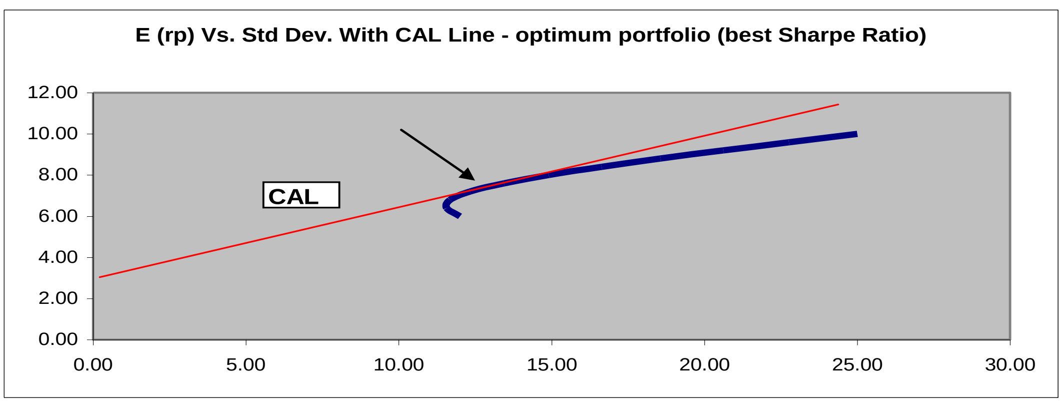
Expected Return – SD combination for any individual asset end-up inside the efficient frontier, because single-asset portfolios are inefficient (are not efficiently diversified)



STEP 2:
Determine the optimal portfolio of risky assets by finding the portfolio that supports the steepest CAL (Risky free return introduced)

Risky free assets – using the current Risk Free Rate, we search for CAL with the highest Sharpe Ratio





STEP 3:
Choose an appropriate complete portfolio based on the investors risk appetite (risk aversion) by mixing the Rf Asset with the optimal risky portfolio.

> *Choose the appropriate optimal risky portfolio (o) above T-bills – Separation Property step - RISK AVERSE comes in play in this step – when selected the desire point of the CAL. More risk averse clients will invest in the risk-free asset and less in the optimal risky portfolio O.*

Chapter 6 - Continued

SINGLE FACTOR ASSET MARKET

Distinction between Systematic and firm-specific Risk. Systematic is largely macroeconomic affecting all securities which firm-specific risk factors affect only one particular firm or, perhaps, its industry.

FACTOR MODELS are structural models designed to estimate these two components of risk for particular security or portfolio.

CAPM – introduction

To construct the efficient frontier from the universe of 100 securities we need 100 expected returns, 100 variances and 100 * 99/2 = 4,950 covariance…. More for more securities…..

ROR in excess of risk free rate (Premium)

Ri = ri – rf

Ri = E (ri) + Bi . M + ei

E (ri) = Expected Excess (Premium)(
Bi = Beta relationship to the industry / market
M = Macroeconomic surprises
Ei = Firm specific events (unanticipated impact)

Dell stock is expected to be 9.0% with beta of 1.2x (every 1.0% move in the market, Dell moves 1.2%, then,

R dell = 9.0% + 1.2. M + e i

R dell = 9.0% + 1.2 * 2% + 0

R dell = 9.0% + 2.4% = 11.4%

CHAPTER 7

CAPITAL ASSET PRICING MODEL (CAPM) AND ARBITRAGE THEORY

CAPM
The model that predicts the relationship between the risk and equilibrium expected returns on risky assets

Unrealistic World
1. Investment Cannot affect process by their individual trades (Perfect Competition)
2. All investors have identical Holding Period
3. Investors form a portfolio of stocks and bonds
4. No taxes / fees
5. Everyone is seeking efficient frontier portfolio
6. Analysis is the same across the board.

EQUILIBRIUM IN SECURITY MARKETS

Market Portfolio (M) is efficient frontier / optimal Risky portfolio

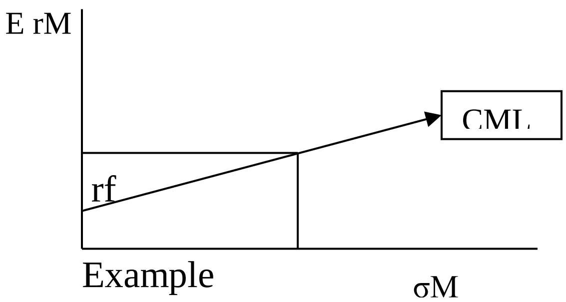
Risk Premium on the market portfolio will be proportional to the variance of the

Mathematically:

$E(r_M) - r_f = A \cdot \sigma_M^2$

σ_M = Standard Deviation of the Return of the Market portfolio
A = Scale Factor representing the degree of risk Aversion

CAPM implies that a passive strategy using CML as optimal CAL is a powerful alternate to an active strategy,



Example

Rf = 5%

Risk Aversion (A) = 2

Standard Deviation of the Market portfolio (M) = 20%

Then E (rM) - rf = A . σM^2

E (rM) = rf – A. σM^2

E (rM) = 0.05 + 0.08 = 13.0%

At A = 3, then 12.0% + 5% = 17%

Historical: S&P had 8.5% Risk Premium with 20% Standard Deviation

$$E(rs) = rf + b \cdot p + e$$

Ri = ri – rf   Excess Return

Ri = E (ri) + Bi . M + ei

REAL WORLD

Let's use it for DCF analysis on a private company – equity analysis – Alexandria Hotel

From Chapter 7 and Instructor's notes - Review

5 TECHNICAL RISK RATIOS – FOR PORTFOLIO MANAGEMENT:

1. **Seeking Alpha** (A measurable way to gauge a manager's ability to outperform the market - Alpha > the Market Return – This will be discussed later in the next LECTURE.
2. **Calculating Beta** (Volatility compared to Market)
3. **Standard Deviation**: Difference / Variation or Deviation from the mean return
4. **R-squared** – statistical measurement that represents % of fund or security 's movement that can be explained by movement in the market benchmarked (S&P 500) scale 0-100% (85 or higher – beta is valid, less than 70, the Beta is not that important – (To be discussed in the next LECTURE)
5. **Sharpe Ratio**: Relationship between Premium Return (Rf – Ri) and Risk (standard deviation).

1. CALCULATING BETA COEFFICIENTS

- The CAPM is an *ex ante* model, which means that all of the variables represent before-the-fact, *expected* values. In particular, the beta coefficient used in the SML equation should reflect the expected volatility of a given stock's return versus the return on the market during some *future* period. However, people generally calculate betas using data from some *past* period, and then assume that the stock's relative volatility will be the same in the future as it was in the past.

- To illustrate how betas are calculated, consider Figure 5A-1. The data at the bottom of the figure show the historical realized returns for Stock J and for the market over the last five years. The data points have been plotted on the scatter diagram, and a regression line has been drawn. If all the data points had fallen on a straight line, as they did in Figure 5-9 in Chapter 5, it would be easy to draw an accurate line. If they do not, as in Figure 5A-1, then you must fit the line either "by eye" as an approximation or with a calculator.

- Recall what the term *regression line*, or *regression equation*, means: The equation Y= a+ bX + e is the standard form of a simple linear regression. It states that the dependent variable, Y, is equal to a constant, a, plus b times X, where b is the slope coefficient and X is the independent variable, plus an error term, e. Thus, the rate of return on the stock during a given time period (Y) depends on what happens to the general stock market, which is measured by X =kM.

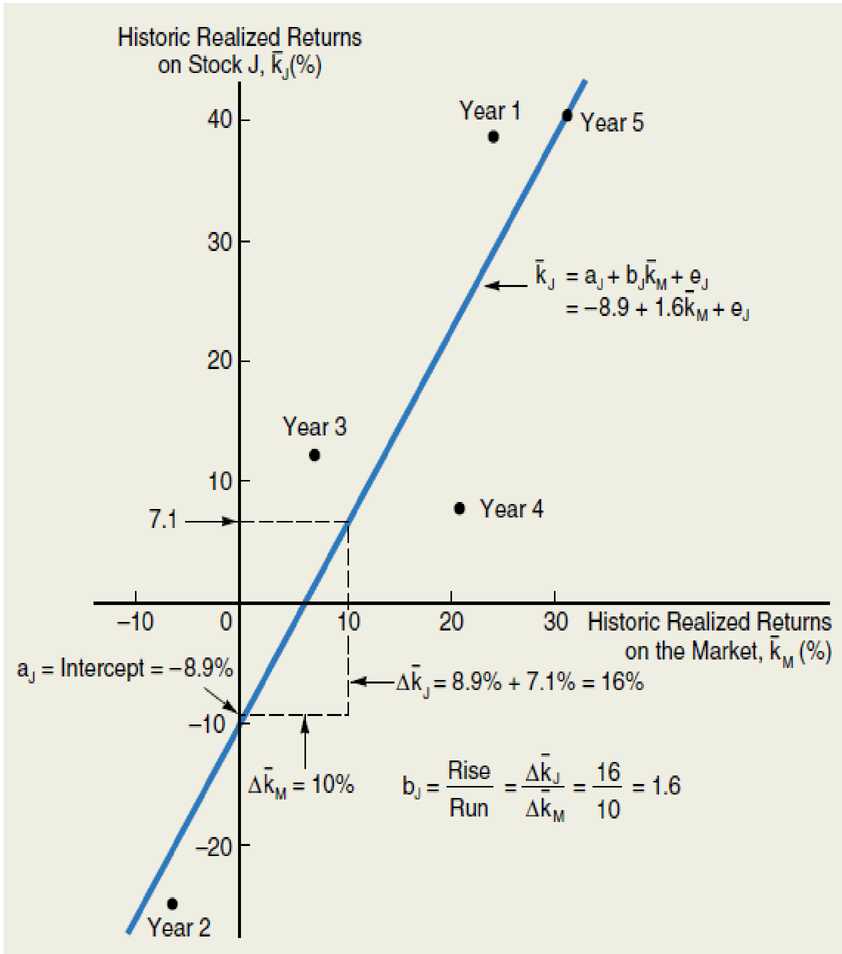
- Once the data have been plotted and the regression line has been drawn on graph paper, we can estimate its intercept and slope, the a and b values in Y = a + bX. The intercept, a, is simply the point where the line cuts the vertical axis. The slope coefficient, b, can be estimated by the "rise-over-run" method. This involves calculating the amount by which kJ increases for a given increase in kM. For example, we observe in Figure 5A-1 that kj increases from 8.9 to 7.1 percent (the rise) when kM increases from 0 to 10.0 percent (the run). Thus, b, the beta coefficient, can be measured as follows:

$$b = \text{Beta} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta Y}{\Delta X} = \frac{7.1 - (-8.9)}{10.0 - 0.0} = \frac{16.0}{10.0} = 1.6.$$

- Note that rise over run is a ratio, and it would be the same if measured using any two arbitrarily selected points on the line. The regression line equation enables us to predict a rate of return for Stock J, given a value of kM. For example, if kM = 15%, we would predict kJ = 8.9% + 1.6(15%) = 15.1%. However, the actual return would probably differ from the predicted return. This deviation is the error term, eJ, for the year, and it varies randomly from year to year depending on company-specific factors. Note, though, that the higher the correlation coefficient, the closer the points lie to the regression line, and the smaller the errors.

- In actual practice, monthly, rather than annual, returns are generally used for Kj and kM, and five years of data are often employed; thus, there would be 5 x 12 = 60 data points on the scatter diagram. Also, in practice one would use the *least squares method* for finding the regression coefficients a and b. This procedure minimizes the squared values of the error terms, and it is discussed in statistics courses.

FIGURE 5A-1 Calculating Beta Coefficients



YEAR	MARKET ($\bar{k}_M$)	STOCK J ($\bar{k}_J$)
1	23.8%	38.6%
2	(7.2)	(24.7)
3	6.6	12.3
4	20.5	8.2
5	30.6	40.1
Average $\bar{k}$	14.9%	14.9%
$\sigma_{\bar{k}}$	15.1%	26.5%

Statistics Worksheet

A	B	C	D	E	F	G	H	I	J	K

Calculating Beta Coefficient

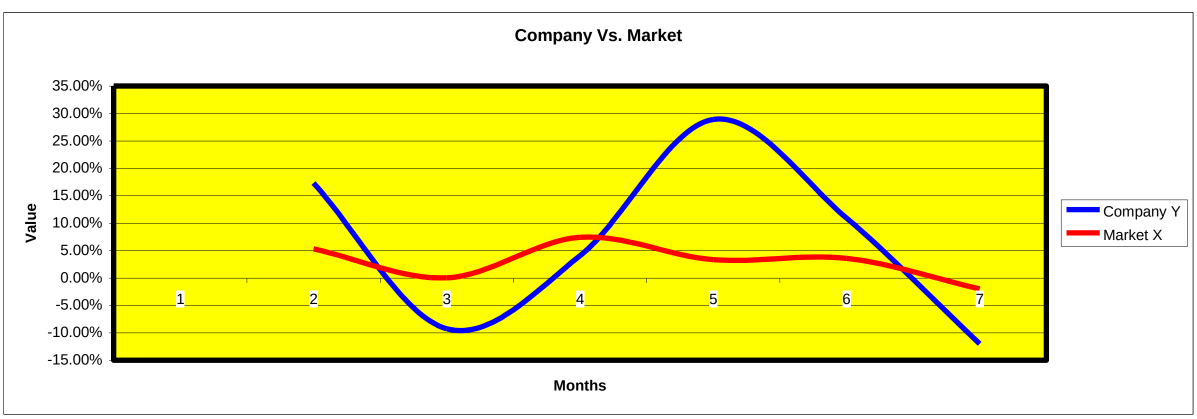
7-month Data

	Day	Starwood Hotel Stock Prices	S&P500 Index	Starwood Change HPR	S&P500 Change HPR
8	30-Apr	20.86	872.81		
9	29-May	24.47	919.14	17.31%	5.31%
10	30-Jun	22.20	919.32	-9.28%	0.02%
11	31-Jul	23.10	987.48	4.05%	7.41%
12	31-Aug	29.78	1020.62	28.92%	3.36%
13	30-Sep	33.03	1057.08	10.91%	3.57%
14	30-Oct	29.06	1036.19	-12.02%	-1.98%
15					
16					
17					
18					

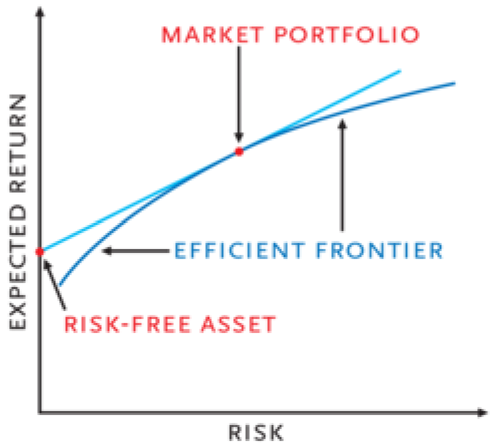
		Dependent Starwood Company Y	Independent S&P Market X	E (Y - Avg Y)	F (X - Avg X)	E x F	F^2	Beta (Slope)
21	30-Apr							
22	29-May	17.31%	5.31%	0.10657	0.02359	0.00251	0.00056	
23	30-Jun	-9.28%	0.02%	-0.15926	-0.02929	0.00467	0.00086	
24	31-Jul	4.05%	7.41%	-0.02595	0.04465	-0.00116	0.00199	
25	31-Aug	28.92%	3.36%	0.22269	0.00407	0.00091	0.00002	
26	30-Sep	10.91%	3.57%	0.04264	0.00623	0.00027	0.00004	
27	30-Oct	-12.02%	-1.98%	-0.18669	-0.04925	0.00919	0.00243	
28	Average =	6.65%	2.95%			0.01639	0.00589	2.782408
30	Variance	2.473%	0.118%					
	St. Deviation =	15.726%	3.432%					

$$\frac{\Sigma\,[y - Avg(y)]\ .\ [x - Avg(x)]}{\Sigma\,[x - Avg\ (x)]^{2}} =$$

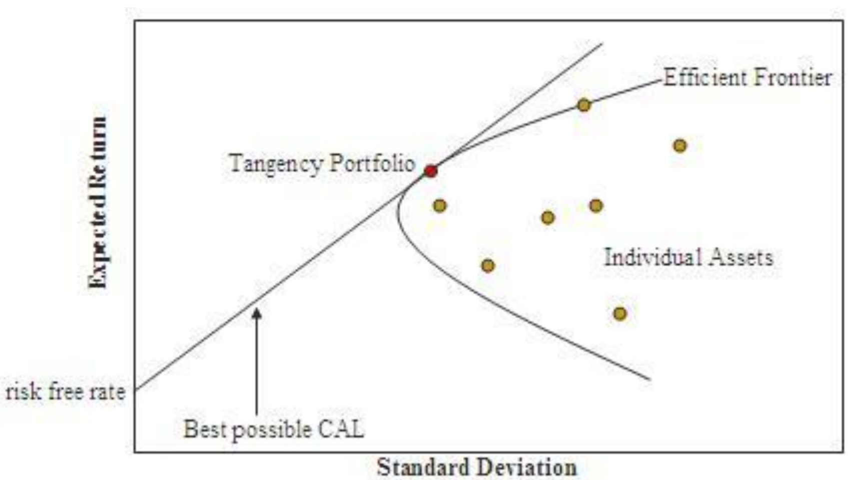
31				
32				
33				
34	Slope (b)=	2.7824	=SLOPE(C21:C27,D21:D27)	Relationship between Dependent Y with Indepent X
35	Forecast =	2.7668	=FORECAST(1,C21:C27,D21:D27)	predicts value of y given a value of x=1%
36	Standard Error =	0.1397	=STEYX(C21:C27,D21:D27)	predicts the standard error y-value for each x in the regression



Modern Portfolio Theory (MPT):



CHALLENGED BY BEHAVIORAL ECONOMICS



Efficient Frontier is the intersection of the Set of Portfolios with Minimum Variance (MVS) and set of portfolios with Maximum Return

Statistics Worksheet

A	B	C	D	E	F	G	H	I	J	K

Calculating Beta Coefficient

7-month Data

Row	Day	Starwood Hotel Stock Prices	S&P500 Index	Starwood Change HPR	S&P500 Change HPR
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15					
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18					

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$$\frac{\Sigma\,[y - Avg(y)]\,.\,[x - Avg(x)]}{\Sigma\,[x - Avg\,(x)]^{2}} =$$

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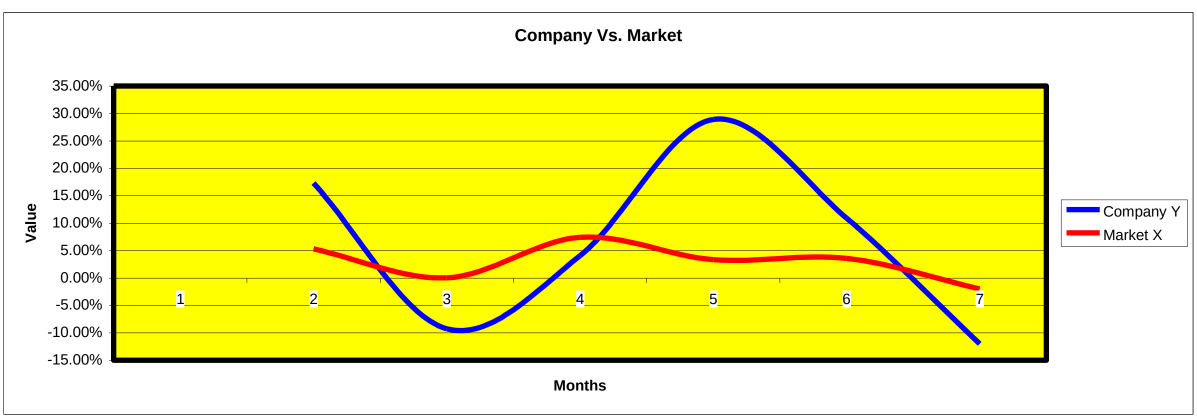

2. CALCULATING STANDARD DEVIATION

A	B	C	D	E	F	G	H
81	**Calculating Standard Deviation**						
82							
83							
84	**7-month Data**						
85	**Day**	**Starwood Hotel Stock Prices Change**			**Variance**		
86	30-Apr						
87	29-May	17.3%			1.14%		
88	30-Jun	-9.3%			2.54%		
89	31-Jul	4.1%			0.07%		
90	31-Aug	28.9%			4.96%		
91	30-Sep	10.9%			0.18%		
92	30-Oct	-12.0%			3.49%		
93	**Average**	**6.65%**		**Variance =**	**2.47%**	=SUM(F115:F121)/C125	
94				**Standard Deviation (Long form) =**	**15.73%**	=SQRT(F122)	
95	**n =**	6	=COUNT(C87:C92)				
96	**n - 1 =**	5	=+C95-1				
97				**Standard Deviation (using Excel) =**	15.73%	=STDEV(C115:C121)	

3. CALCULATING R SQUARE

SUMMARY OUTPUT

Regression Statistics		*Explanation*
Multiple R	0.6072	Square Root of R Square
R Square	0.3687	Low R squared (Beta coefficinet is not reliable)
Adjusted R Square	0.2109	This is used if more than one x variable
Standard Error	0.1397	This is the sample estimate of the standard deviation of the error
Observations	6	Number of observations used in the regression

ANOVA (Analysis of variance) This table splits the sum of the squares into its components

	df	*SS*	*Explanation*	*MS*	*F*	*Significance F*
Regression	1	0.045596541		0.045596541	2.33662503	0.20109
Residual	4	0.078055383	← R^2 = 1- (0.0781/0.1237)	0.019513846		
Total	5	0.123651924	← Total			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Upper 95.0%*
Intercept	-0.015561849	0.078318048	-0.1987007	0.8522	-0.2330	0.2019	-0.2330	0.2019
X Variable 1	2.782407573	1.820229858	1.52860231	0.2011	-2.2714	7.8362	-2.2714	7.8362

4. CALCULATING SHARP RATIO

A	B	C	D
100	**Calculating Sharp Ratio**		
101			
102	Risk Free (rf) =	2.50%	
103	Return =	6.65%	
104	Standard Deviation =	15.73%	
105			
106			
107	**Sharp Ratio**	**0.26**	=+(C132-C131)/C133
108			
109			